(Geo)Technical Booklet No2

Soil undrained shear strength

A journey to the land of undrained failures

A. Pérez Gallardo

FOREWORD

Imagine that you have arrived at a distant land where local engineers were suddenly becoming obsessed with the imminence of undrained ground failures on their ancient well performing embankment dams. Imagine that factors of (un)safety lower than the unit were becoming an ordinary way for engineers to manifest how insecure their structures were, albeit having them standing still before their own eyes, in cases for many decades. The author of this booklet arrived once, not too long ago, at one of such places and, faithful to mankind, started questioning once again everything he believed up until then. To exorcise his demons, he turned one more time to soil mechanics, the only place, he thought, where true beliefs could be built and reexamined what he had been preaching for more than three decades of practice. What have I failed to understand? he asked, contemplating such a fervorous creed on unsafety. Is it possible that those who came before us were so unenlightened? he wondered, while thinking of the voiceless fathers of so many dangerous embankment dams now said to be on the verge of collapse. This booklet compiles the notes that the author took as a kind of catharsis, while working as an invitee geotechnical engineer for the owner of hundreds of dams on that unfamiliar distant land, dispirited to watch a contagious outbreak of fear and doubt on engineering practices. Have his notes as they are and for the reason they were written, a personal quest to accept a new surge of faith in the inevitability of spontaneous undrained ground failures on the soil mechanics´ long-term. An unsuccessful quest, so far.

The universe of technical publishing is full of illuminating sureness and there seems to be little room for not knowing or not understanding. But, not knowing and not understanding may be a positive driving force and deserves, we think, a space like the present collection of (Geo)Technical Booklets, dedicated to a branch of civil engineering that sometimes seems to lack sharable scientism. This is the first edition of the second booklet of the collection, issued on august 2024. While doubts persist on matters of technical nature and as long as there is goodwill to deconstruct them openly, booklets like this one could continue endlessly and the author has already promised a revised version.

The collection of (Geo)Technical Booklets is not peer reviewed, is free of editorial guidance and is completely gratuitous. No rights are reserved and respect for the author’s moral rights should be a matter of concern for the contents’ users, as are the consequences of its misuse. Black ribbon free publications, Lisbon.

1. PREAMBLE

Everyone has superb soil mechanics books on his library shelves, and everyone has studied thousands of elegant articles published in the best technical magazines. This booklet is not intended to match up with none of those, nor the author would aspire to such endeavor. However, reexamining our beliefs frequently requires us to wipe the dust off our own library shelves. On the episode that motivated this writing, things got more complicated than in a routine cleanse, as the author’s library of beliefs was caught by a dust storm. Do soils behave or do soils respond to loads is the question being pursued in this text, using the stability of embankment dam slopes founded on week compressible soils to provide context. We will start from what may be unanimously accepted, although not always exercised, regarding soil undrained shear strength and end up with the dusty dilemma regarding ground undrained failures, a subject that sways back and forth without any firm answer.

2. LABORATORY STRENGTH

2.1. Soft fundaments of soil “behaviour”

For convenience, as later better understood, we will start rejecting the idea that soils behave, as if the same soil responded equally to different loading conditions, depending exclusively on their own state and structure. Let us assume that the same soil responds differently under different load conditions, as a starting point to discuss its various undrained shear strengths.

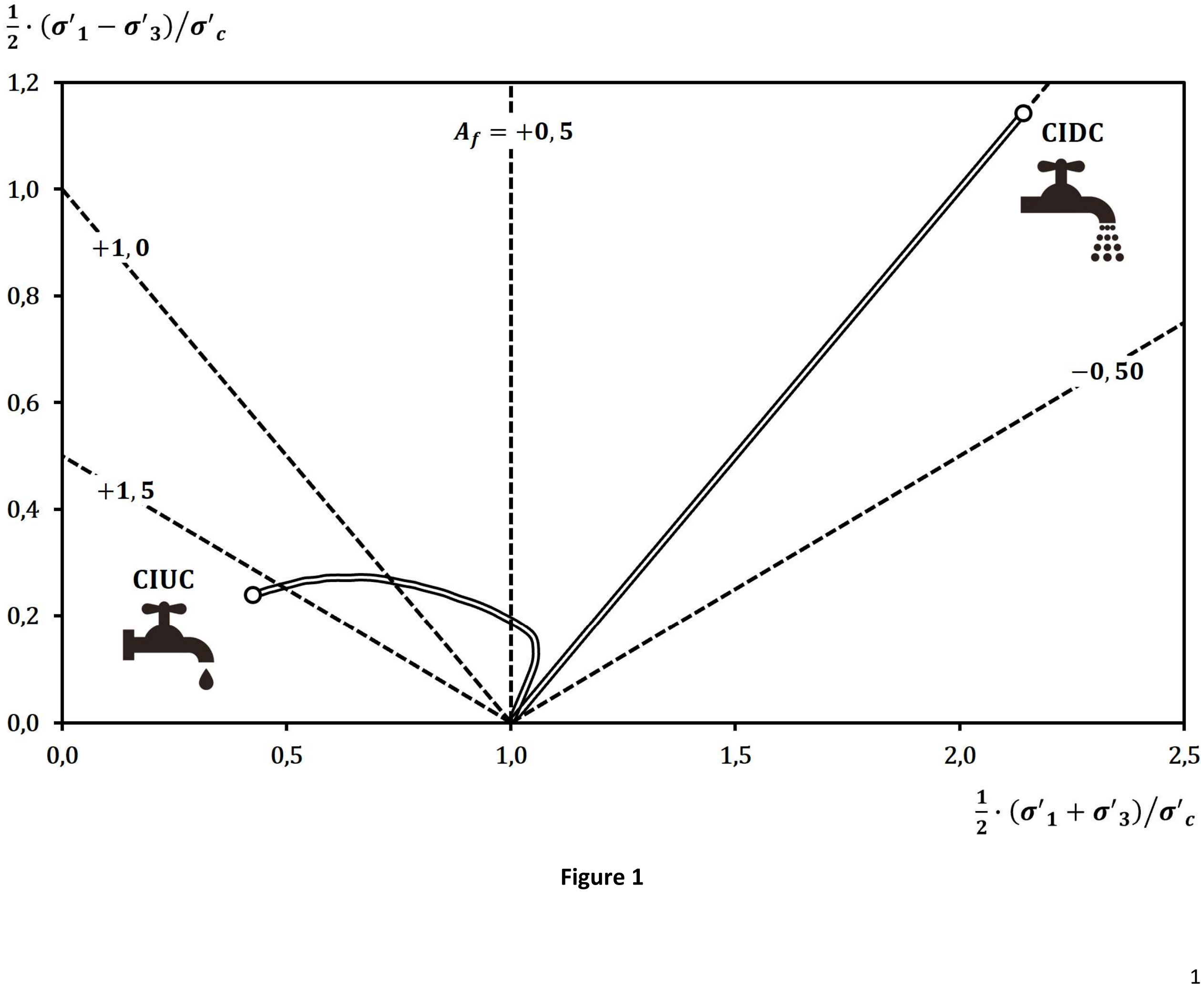

Figure 1

Figure 1, at the end of last page, shows the effective stress path followed by two specimens of the same block sample, both specimens consolidated isotropically to the same stress ($\sigma'_c = \sigma'_{1c} = \sigma'_{3c}$) on a triaxial apparatus. Dashed lines represent the direction of the secant path to failure for different values of Skempton's pore pressure coefficient A. Regardless of the soil's state and structure, our introductory statement is unequivocal in view of this evidence. One soil, two responses. In fact, one soil, infinite responses. The CU stress path corresponds to the ideal condition of blocked drainage, equivalent to excessively fast loading. The CD tests correspond to the ideal condition of free drainage, equivalent to sufficiently slow loading. The soil specimens had no will of their own, they responded accordingly to the applied loading and to the imposed boundary conditions.

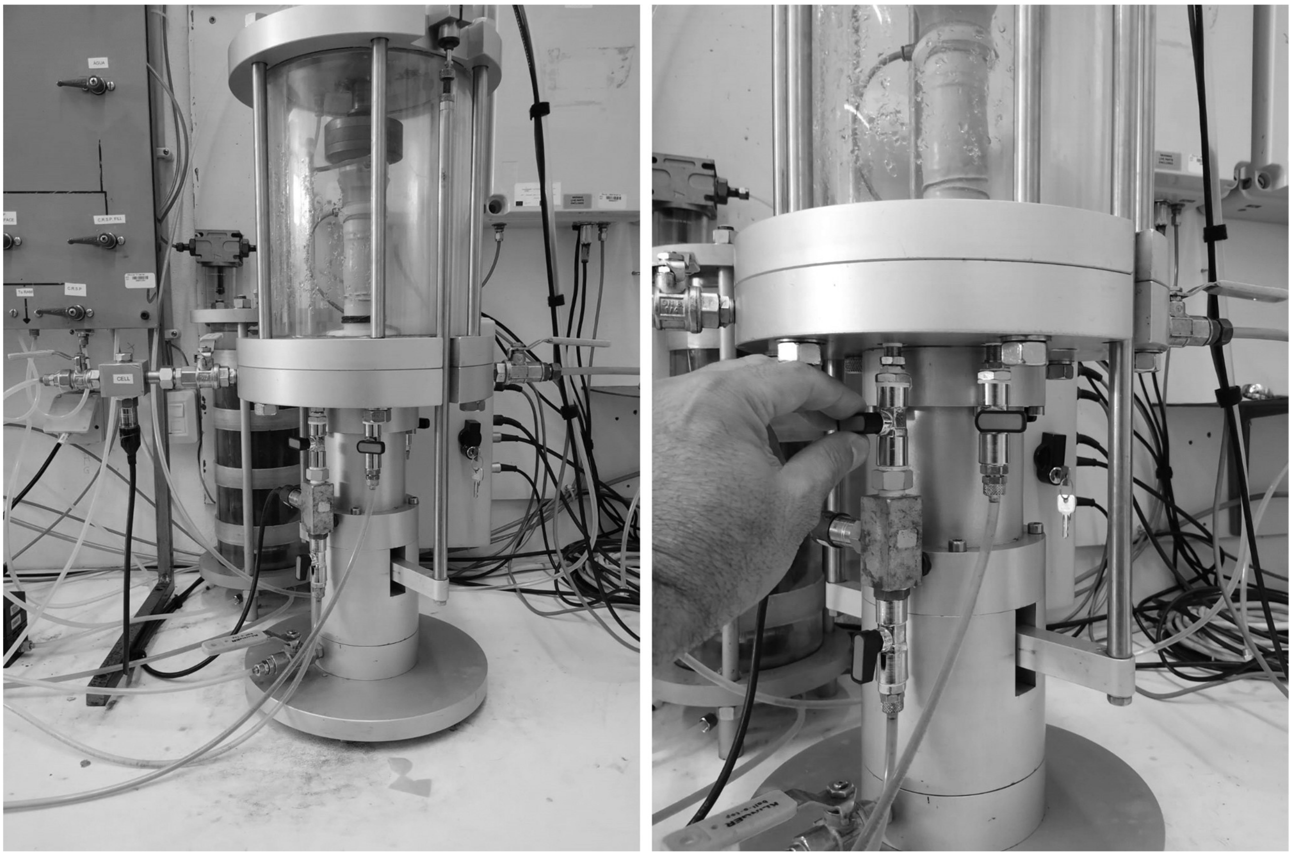

Figure 2

In Figure 2 we can see a triaxial testing apparatus and, on the right-side picture, we can see the laboratory technician literally "closing the tap", that is, establishing the boundary conditions to measure the soil response to a compression load (increase in the axial pressure with radial pressure kept constant) under ideal blocked drainage conditions. The latter loading leads to the mobilisation of soil undrained shear strength, which differs from the drained one by the amount of excess pore water pressures that could not be dissipated along with the application of the external load during the test shearing stage. Undrained responses are not exclusive of soft or loose soils, say contractive soils. Any soil loaded in ideal conditions of prevented drainage will manifest what we know as undrained shear strength, some generating more and some generating less excesses of pore-pressure. Some soils generate positive, and some generate negative excesses of pore water pressures at the instant of failure. What happens along the stress path is surely interesting but not determinant to define the instant of failure (unfilled points in Figure 1) and the soil strength.

For the following discussion around triaxial testing we will be using Skemtpton´s equation for the excess pore pressures Δu generated at failure (index f) by an incremental variation of the maximum $\Delta\sigma_1$ and/or minimum $\Delta\sigma_3$ principal stresses in conditions of false triaxiality on a fully saturated soil specimen (Skempton´s pore pressure coefficient B equal to unity):

$$\begin{aligned}\Delta u &= \Delta\sigma_3 + A_f \cdot (\Delta\sigma_1 - \Delta\sigma_3) = \\ &= \frac{1}{3}\cdot(\Delta\sigma_1 + 2\cdot\Delta\sigma_3) + \frac{1}{3}\cdot\left(3\cdot A_f - 1\right)\cdot(\Delta\sigma_1 - \Delta\sigma_3) = \Delta p + \frac{1}{3}\cdot\left(3\cdot A_f - 1\right)\cdot\Delta q\end{aligned} \quad [01]$$

Where Δp and Δq are the corresponding increments of mean and deviatoric stress. Skemtpton's A coefficient at failure ($A = A_f$) is all what is needed to fully define any secant effective stress path separating the initial and failures states and, as such, the only parameter, alongside with the soil effective angle of shearing resistance φ', required to theoretically derive the different undrained shear strengths that a soil can mobilise under different loading combinations on the triaxial apparatus.

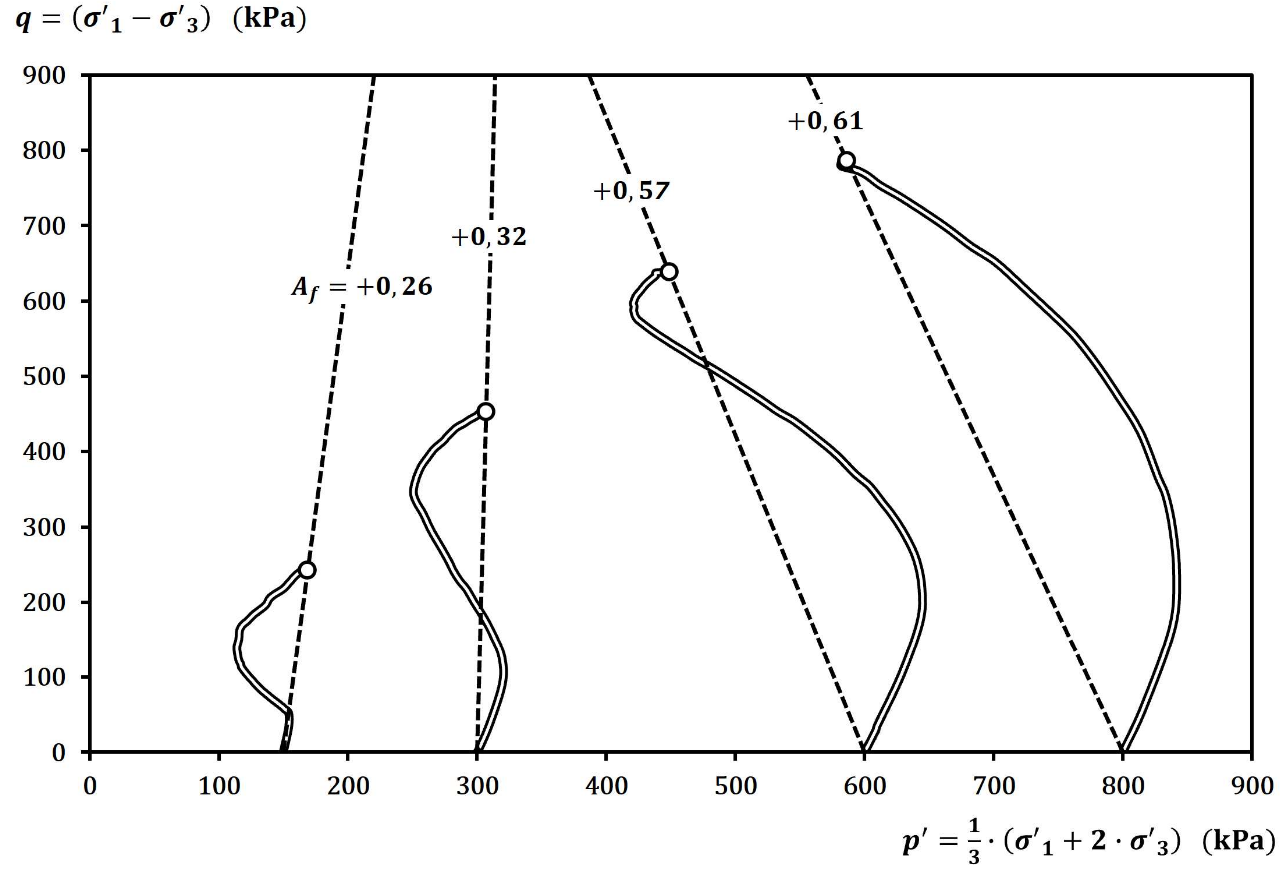

Figure 3

Figure 3 shows, on Cambridge p-q stress diagram, the effective stress path of four specimens of a lightly overconsolidated natural clayey soil under an undrained axial compression (triaxial CIUC). As stated before, the secant stress path, from the respective isotropic consolidation stress up to the instant of failure (unfilled dots), assumed to occur at the maximum deviatoric stress, is fully defined by the Skempton's pore water coefficient A_f. We give this example at this stage with another aim in mind. The intact sample subjected to triaxial testing was a natural very stiff (consistency index over 1) high plasticity clay collected from a dam sitting on a tropical residual soil profile and was arbitrarily considered as "behaving always in a drained manner", perhaps because it was not a soft soil. If the "tap is closed" on the laboratory, what we observe on the figure is an undrained response. In fact, not one but, on the figure, four responses. We will further discuss what happens on the field, where taps do not exist.

To finalise this section, it is better to define what is, according to Tresca failure criteria, the undrained shear strength s_u:

$$s_u = \frac{q_f}{2} = \frac{(\sigma'_1 - \sigma'_3)_f}{2} \qquad [02]$$

With:

$$q_f = \max(q) \qquad [03]$$

From a CIUC triaxial test, the deviatoric stress at failure q_f is simply read on the ordinate axis of the graph on Figure 3 and is an uncontested direct measurement. The measured shear strength for that loading is $(s_u)_{CIUC}$ and the index is important to distinguish it from any other undrained shear strength. Strength of a material refers to its ability to withstand an applied load without failure or plastic deformation. Failure of a material refers to its loss of load carrying capacity. In Figure 3, for the soil specimen isotropically consolidated to a stress of 150kPa, the triaxial CIUC undrained shear strength is exactly $(s_u)_{CIUC} = 121$KPa, not half, nor a third of it. The undrained shear ratio, using the consolidation stress as a normalizing parameter, is exactly $(s_u/\sigma'_c)_{CIUC} = 0{,}80$, not the result of any philosophical discussion about how tortuous the stress path was until then.

2.2. Soil multiple "behaviours" in a soft overview

Still at the laboratory discussing what we can directly measure, concerning soil shearing resistance. Referring to Figure 4, on next page, there are four total stress paths typically used on the triaxial testing apparatus that, to some extent, replicate the real loading conditions of most geotechnical problems. We denote $\Delta\sigma_a$ as the axial pressure increment and $\Delta\sigma_r$ the radial stress increment and make the necessary correspondences with the principal stress increments so that equation 01 holds true for any type of test. Also, to distinguish between the Cambridge and the MIT stress diagrams, we will be denoting the stress invariants used on the latter as $s' = (\sigma'_1 + \sigma'_3)/2$ and $t' = (\sigma'_1 - \sigma'_3)/\,2$. From those four total stress paths, two of them, a compressive failure by axial compression (AE) and an extension failure by lateral compression (LC) may be faced as the primary bounds of the shearing strength governing the failure of a weak embankment foundation. As this booklet has been written in the context of embankment dam safety, both referred load paths deserve better attention.

However, we may show first that the effective stress path followed on an axial compression test (AC) is the same as the one followed on a lateral extension test (LE), both leading to compression failures. The same happens with the two stress paths leading to extension failures. We have stated before that the secant effective stress path to failure is fully defined by Skempton´s pore pressure coefficient A at failure:

$$\left(\frac{\Delta q}{\Delta p'}\right)_f = \frac{\Delta(\sigma_1 - \sigma_3)_f}{\Delta(\sigma_1 + \sigma_3)_f - 2 \cdot \Delta u_f} \tag{04}$$

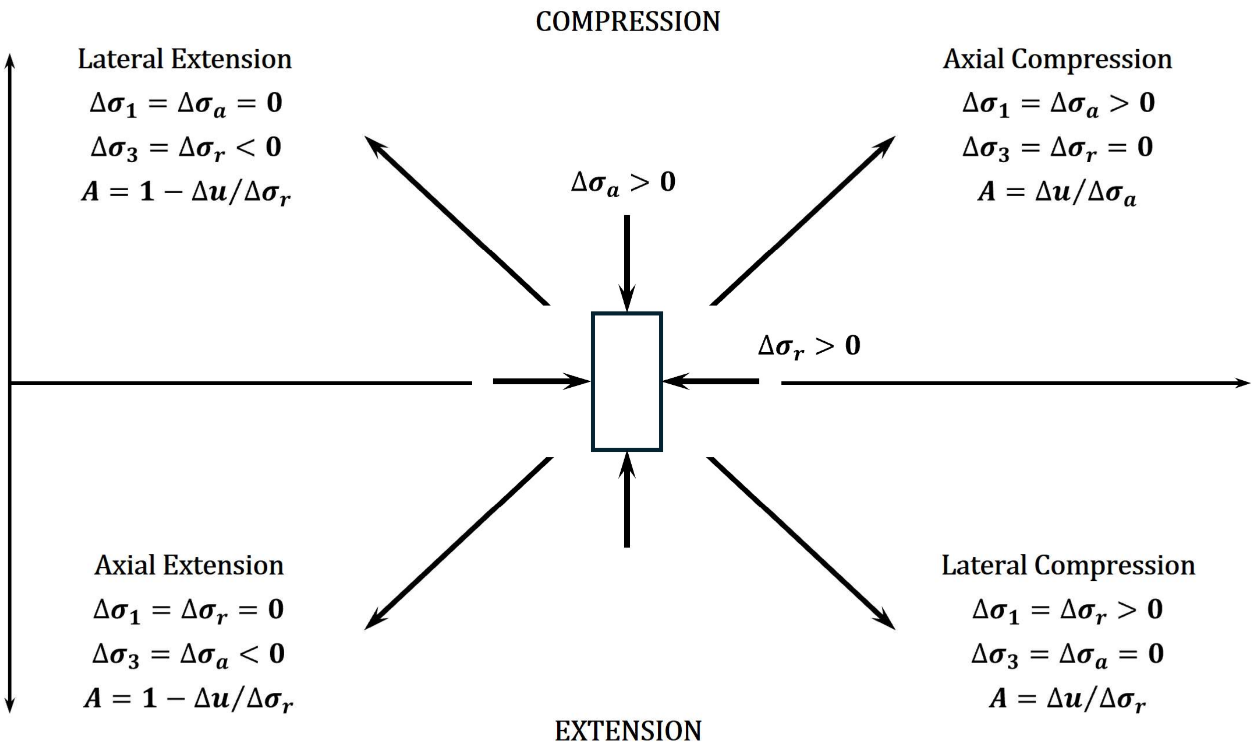
COMPRESSION
Lateral Extension
$\Delta\sigma_1 = \Delta\sigma_a = 0$
$\Delta\sigma_3 = \Delta\sigma_r < 0$
$A = 1 - \Delta u/\Delta\sigma_r$
Axial Compression
$\Delta\sigma_1 = \Delta\sigma_a > 0$
$\Delta\sigma_3 = \Delta\sigma_r = 0$
$A = \Delta u/\Delta\sigma_a$
$\Delta\sigma_a > 0$
$\Delta\sigma_r > 0$
Axial Extension
$\Delta\sigma_1 = \Delta\sigma_r = 0$
$\Delta\sigma_3 = \Delta\sigma_a < 0$
$A = 1 - \Delta u/\Delta\sigma_r$
Lateral Compression
$\Delta\sigma_1 = \Delta\sigma_r > 0$
$\Delta\sigma_3 = \Delta\sigma_a = 0$
$A = \Delta u/\Delta\sigma_r$
EXTENSION

Figure 4

For the case of an axial compression failure, we have $\Delta\sigma_1 = +\Delta\sigma_a$ and $\Delta\sigma_3 = \Delta\sigma_r = 0$. For the case of a lateral extension failure, we have $\Delta\sigma_1 = \Delta\sigma_a = 0$ and $\Delta\sigma_3 = -\Delta\sigma_r$. Using Skempton´s equation 01, we have for the first case:

$$A_{f,AC} = \Delta u_f / \Delta\sigma_{1,f} \tag{05}$$

And for the second one:

$$A_{f,LE} = 1 - \Delta u_f / \Delta\sigma_{1,f} \tag{06}$$

Substituting each of them on equation 03:

$$\left(\frac{\Delta q}{\Delta p'}\right)_{f,AC} = \frac{3}{1 - 3 \cdot A_{f,AC}} \equiv \left(\frac{\Delta q}{\Delta p'}\right)_{f,LE} = \frac{3}{1 - 3 \cdot A_{f,LE}} \tag{07}$$

Which are equal because it happens that $A_{f,AC} = A_{f,LE}$. The same way, for extension failures, $A_{f,LC} = A_{f,AE}$.

Table 1

	$\Delta u = B \cdot \left(\Delta\sigma_3 + A \cdot (\Delta\sigma_1 - \Delta\sigma_3)\right)$		
	σ_r, σ_a	s, t	p, q
CIUC (AC)	$\Delta\sigma_3 = \Delta\sigma_r = 0\ , \Delta\sigma_1 = +\Delta\sigma_a = \ , \Delta u = A \cdot \Delta\sigma_a$		
	$\Delta\sigma'_r = \Delta\sigma_r - \Delta u = -A \cdot \Delta\sigma_a$	$\Delta s' = \frac{\Delta\sigma_a}{2} - \Delta u = \frac{1}{2} \cdot (1 - 2 \cdot A) \cdot \Delta\sigma_a$	$\Delta p' = \frac{\Delta\sigma_a}{3} - \Delta u = \frac{1}{3} \cdot (1 - 3 \cdot A) \cdot \Delta\sigma_a$
	$\Delta\sigma'_a = \Delta\sigma_a - \Delta u = (1 - A) \cdot \Delta\sigma_a$	$\Delta t' = \frac{\Delta\sigma_a}{2}$	$\Delta q' = \Delta\sigma_a$
	$\frac{\Delta\sigma'_a}{\Delta\sigma'_r} = \frac{-(1 - A)}{A}$	$\frac{\Delta t'}{\Delta s'} = \frac{1}{1 - 2 \cdot A}$	$\frac{\Delta q'}{\Delta p'} = \frac{3}{1 - 3 \cdot A}$
CIUE (LC)	$\Delta\sigma_3 = \Delta\sigma_a = 0\ , \Delta\sigma_1 = +\Delta\sigma_r\ , \Delta u = A \cdot \Delta\sigma_r$		
	$\Delta\sigma'_r = \Delta\sigma_r - \Delta u = (1 - A) \cdot \Delta\sigma_r$	$\Delta s' = \frac{\Delta\sigma_r}{2} - \Delta u = \frac{1}{2} \cdot (1 - 2 \cdot A) \cdot \Delta\sigma_r$	$\Delta p' = \frac{\Delta\sigma_r}{3} - \Delta u = \frac{1}{3} \cdot (1 - 3 \cdot A) \cdot \Delta\sigma_r$
	$\Delta\sigma'_a = \Delta\sigma_a - \Delta u = -A \cdot \Delta\sigma_r$	$\Delta t' = \frac{\Delta\sigma_r}{2}$	$\Delta q' = \Delta\sigma_r$
	$\frac{\Delta\sigma'_a}{\Delta\sigma'_r} = \frac{-A}{(1 - A)}$	$\frac{\Delta t'}{\Delta s'} = \frac{1}{1 - 2 \cdot A}$	$\frac{\Delta q'}{\Delta p'} = \frac{3}{1 - 3 \cdot A}$

Table 1 on the previous page summarizes the relevant relationships between stress increments and excess pore pressures for the two types of tests of more relevance to the embankment dam slope stability problem, expressed in different stress spaces. There is a unicity on the effective stress paths to failure, but unfortunately there is not equality in values for compression and extension failure types. The value of the pore pressure coefficient A_f is different for each failure mode and there is no theory capable of explaining why beyond empiricism. If the amount of excess pore water pressures depends on the loading, the undrained shear strength of soil also is load dependent. Later we will be on the horns of a dilemma because of mixing strength and loading.

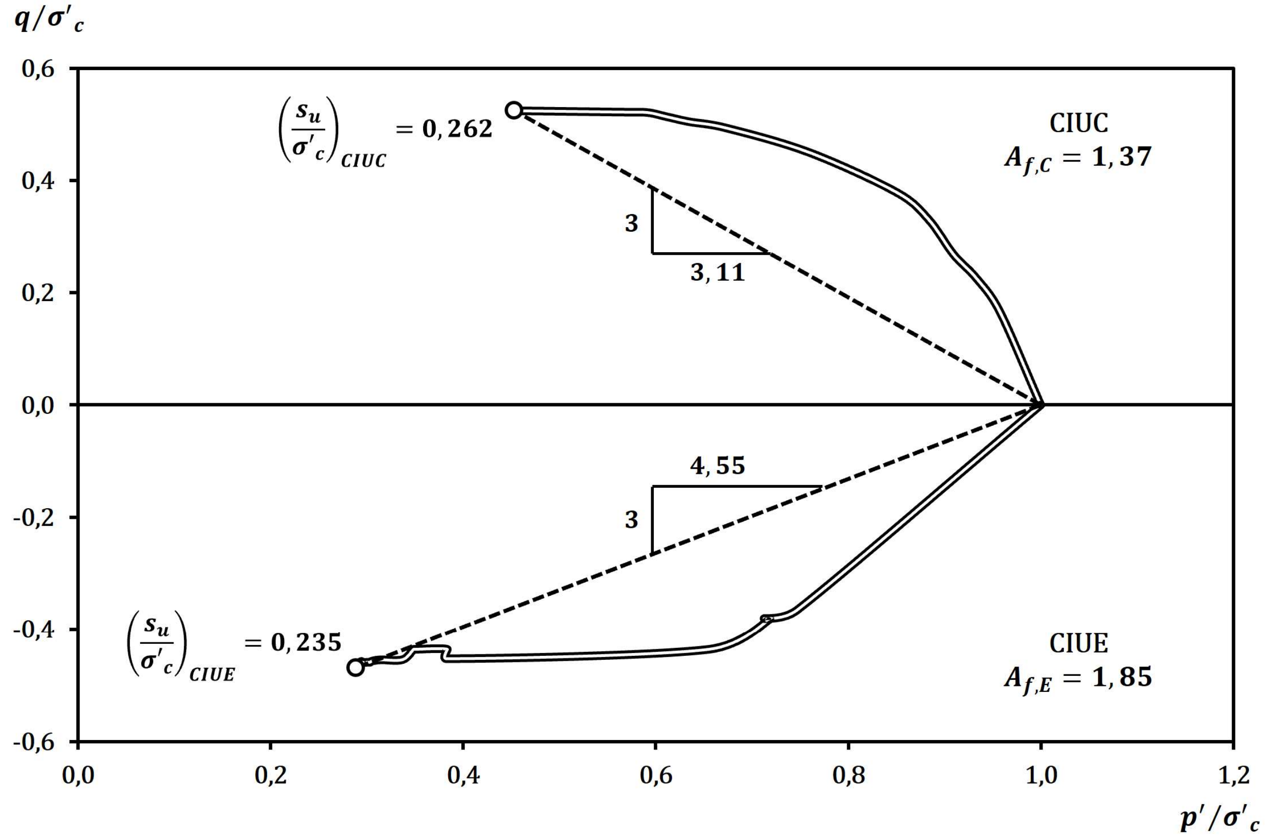

Figure 5

A real case of remolded clay specimens, isotropically consolidated and taken to failure either by an axial compression (CIUC triaxial test) or by an axial extension (CIUE triaxial test) are shown in Figure 5. This example demonstrates that, despite the unicity of the effective stress path, the magnitude of excess pore pressure generated on an extension failure is greater than that generated on a compression failure. Correspondingly, the undrained shear strength on the former case is lower than on the latter one. Differences are not neglectable. In fact, they may be mortiferous.

The diagram presented in Figure 6, at the beginning of the next page, and some work around trigonometric relationships allow us to derive equations for the undrained shear strength, s_u, compatible with the Mohr-Coulomb failure criteria in terms of effective stress parameters. For a general case deduction, it is assumed that the effective strength linear failure envelope comprises a cohesion intercept, c'. As well, we will be starting from the case where specimens are anisotropically consolidated (CAUC or CAUE tests), being K the ratio between the minimum, σ'_{3c}, and the maximum, σ'_{1c}, principal stresses at the end of the test consolidation stage.

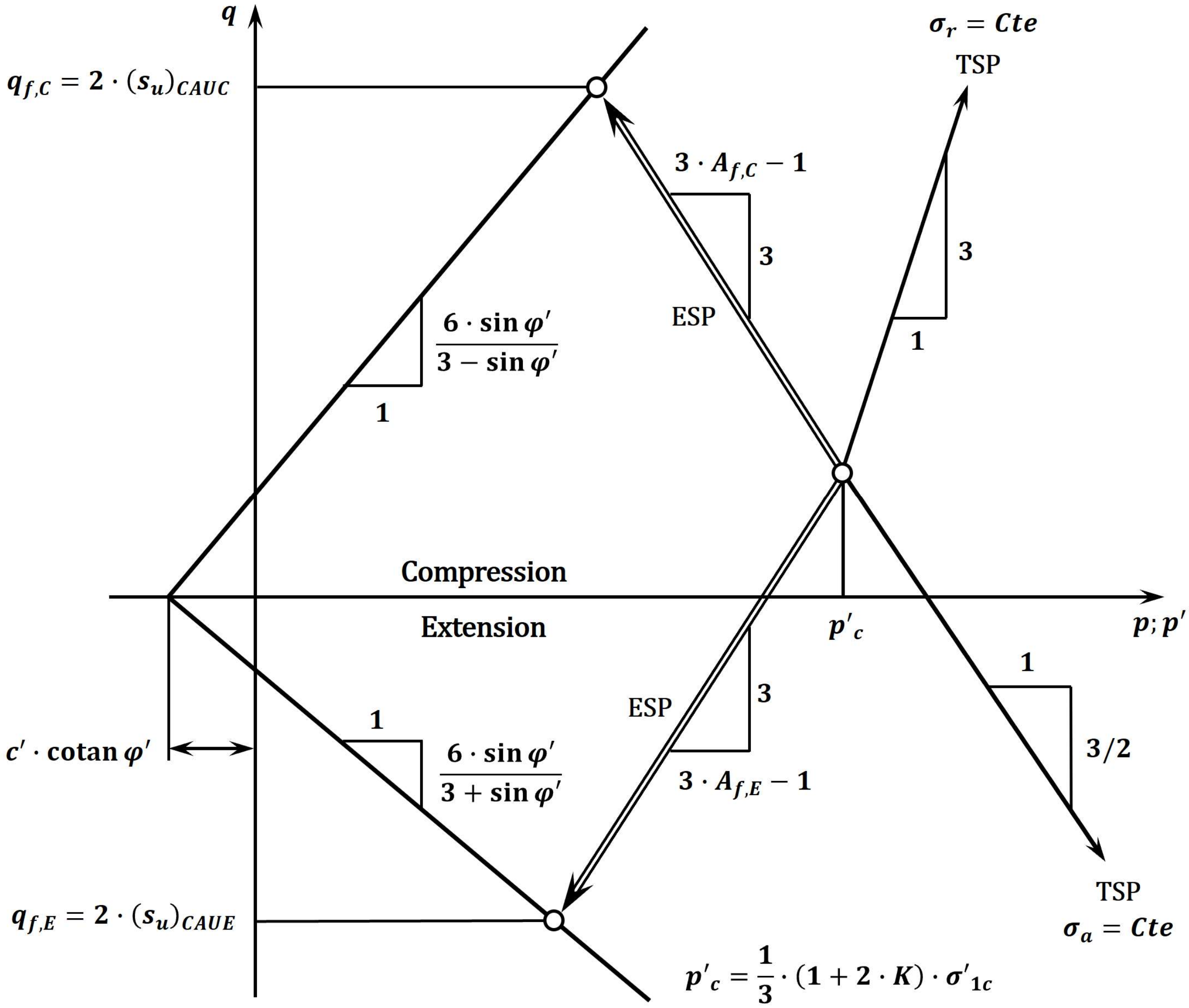
q
$q_{f,C} = 2 \cdot (s_u)_{CAUC}$
$\sigma_r = Cte$
TSP
$3 \cdot A_{f,C} - 1$
3
ESP
3
1
$\frac{6 \cdot \sin\varphi'}{3 - \sin\varphi'}$
1
Compression
Extension
p'_c
$p; p'$
1
3/2
ESP
3
$3 \cdot A_{f,E} - 1$
1
$\frac{6 \cdot \sin\varphi'}{3 + \sin\varphi'}$
$c' \cdot \text{cotan}\,\varphi'$
$q_{f,E} = 2 \cdot (s_u)_{CAUE}$
TSP
$\sigma_a = Cte$
$p'_c = \frac{1}{3} \cdot (1 + 2 \cdot K) \cdot \sigma'_{1c}$

Figure 6

For the compression failure (CAUC) we have:

$$(s_u)_{CAUC} = \frac{c' \cdot \cos\varphi' + \sigma'_{1c} \cdot \left(K + (1 - K) \cdot A_{f,C}\right) \cdot \sin\varphi'}{1 + \left(2 \cdot A_{f,C} - 1\right) \cdot \sin\varphi'} \qquad [08]$$

And for the extension failure (CAUE):

$$(s_u)_{CAUE} = \frac{c' \cdot \cos\varphi' + 1/2 \cdot \sigma'_{1c} \cdot \left((1 + K) - \left(2 \cdot A_{f,E} - 1/3\right) \cdot (1 - K)\right) \cdot \sin\varphi'}{1 + \left(2 \cdot A_{f,E} - 1/3\right) \cdot \sin\varphi'} \qquad [09]$$

We stress again that the undrained shear strength depends on the loading. Now we have just seen that undrained shear strength also depends on the initial state of stress. So, let us remember to identify in our geotechnical reports where the values being stated were taken from. The greater undrained shear strength that can be measured is that obtained on the CIUC triaxial test, which is routinely done in all commercial laboratories.

Isotropic consolidation is a particular case of the anisotropic one, and we may simply obtain the respective equation for the former by setting the consolidation stress ratio $K = \sigma'_{3c}/\sigma'_{1c}$ to unity in equations 08 or 09. The well-known equation for the CIUC undrained shear strength ratio of a cohesionless soil is:

$$\left(\frac{s_u}{\sigma'_{1c}}\right)_{CIUC} = \frac{\sin\varphi'}{1 + \left(2 \cdot A_{f,C} - 1\right) \cdot \sin\varphi'} \tag{10}$$

The CIUC triaxial test is our reference test and, if we could establish interrelations among the pore water pressure coefficient for each case, the undrained shear strength mobilised by a normally consolidated soil under different failures modes could be related to that mobilised on the reference test. Introducing here the first element α_1 of a chain product factor, latter on explained, the influence of the consolidation stress ratio on a compression failure can be expressed as:

$$\alpha_{11} = \frac{(s_u)_{CK_0UC}}{(s_u)_{CIUC}} = \frac{1 + \left(2 \cdot \left(A_f\right)_{CIUC} - 1\right) \cdot \sin\varphi'}{1 + \left(2 \cdot \left(A_f\right)_{CK_0UC} - 1\right) \cdot \sin\varphi'} \cdot \left(K_0 + (1 - K_0) \cdot \left(A_f\right)_{CK_0UC}\right) \tag{11}$$

Where K_0 is the coefficient of earth pressures at rest. Similarly, for an extension failure:

$$\begin{aligned}\alpha_{12} = \frac{(s_u)_{CK_0UE}}{(s_u)_{CIUC}} &= \frac{1 + \left(2 \cdot \left(A_f\right)_{CIUC} - 1\right) \cdot \sin\varphi'}{1 + \left(2 \cdot \left(A_f\right)_{CK_0UE} - \frac{1}{3}\right) \cdot \sin\varphi'} \times \\ &\times \frac{1}{2} \cdot \left((1 + K_0) - \left(2 \cdot \left(A_f\right)_{CK_0UE} - 1/3\right) \cdot (1 - K_0)\right)\end{aligned} \tag{12}$$

Both expressions assume, even if as an approximation, that the angle of shearing resistance φ' is a soil intrinsic property, which remains constant independently of the loading type. Not even that is more than a convenient approximation. The same way, as discussed later, we will assume that the effect of the stress history may be treated independently of the effect of loading and afterwards superimposed to it. However, in fact, both effects interfere with each other, and one may exacerbate the other.

Unless an extended laboratory testing program is put into practice for each study case, interrelationships between the different pore water coefficients can emerge empirically from a database of tests done in similar soils. As an example of an empirical interrelationship, in Figure 7, on the next page, the results of 63 pairs of pore pressure coefficients from compression, $A_{f,CIUC}$, and extension, $A_{f,CIUE}$, triaxial tests, each pair performed on isotropically consolidated specimens of the same soil, are plotted together and a possible expression to relate them is advanced. The dispersion of results is remarkable, but consistently shows that the excess pore pressures generated on an extension failure can be significantly greater than the ones generated on a compressive failure.

Among the pairs of pore pressure coefficients shown in Figure 6, some were obtained on mechanically over-consolidated specimens and correspond to the values on lower range. To some extent, the above referred interference of over-consolidation on the interrelationship seems to be slight and a constant difference seems to be sufficiently good for practical applications.

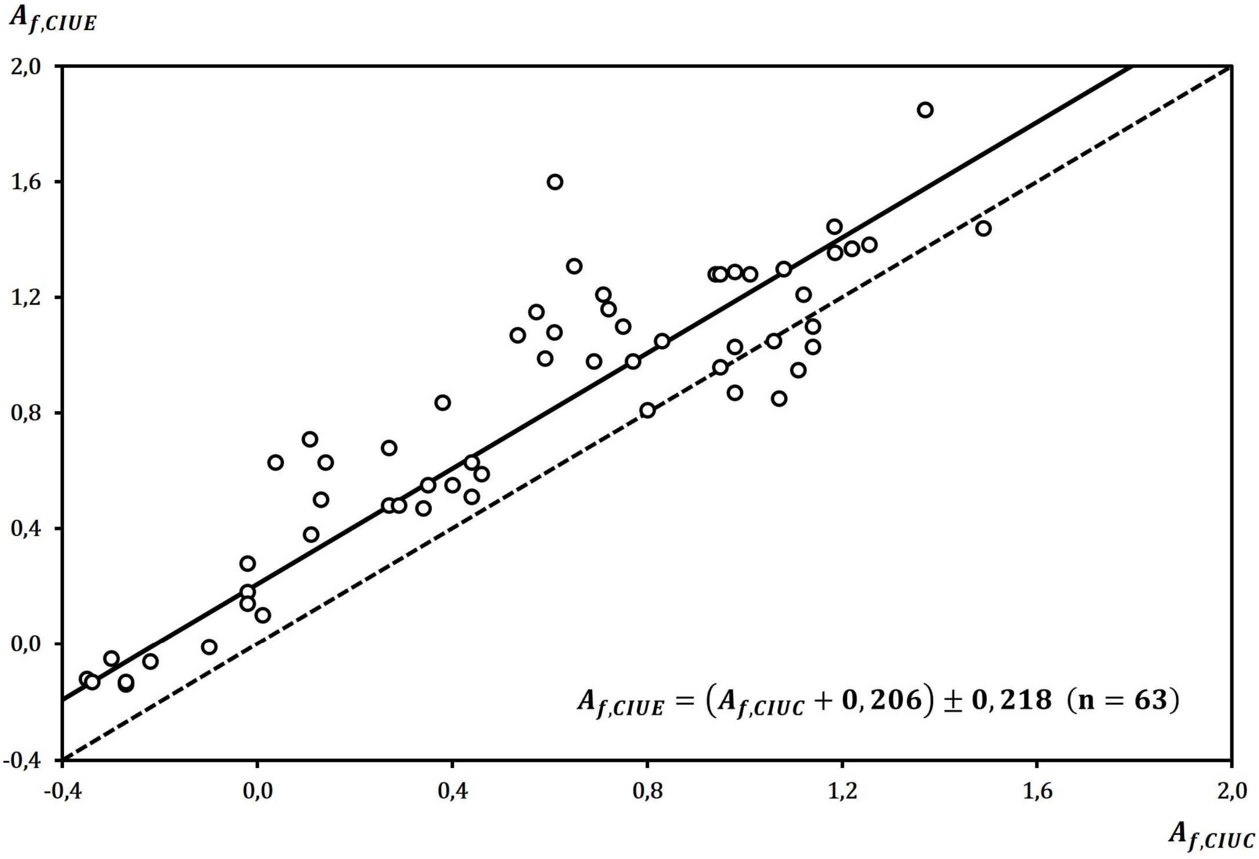

Figure 7

It is worth mentioning at this stage that, if the Tresca failure criteria held true, the direction of the failure planes would form an angle of $\pi/4$ with the direction of the principal stresses, but such realisation is not observed on experiments. The failure planes, according to Mohr-Coulomb failure criteria, form an angle of $\pi/4 - \varphi'/2$ with the direction of the maximum principal stresses and are in better agreement with those observed in tests. The undrained shear strength according to the latter criteria, that we will denote as c_u, relates to the former one:

$$c_u = \mathrm{s}_u \cdot \cos\varphi' \qquad [13]$$

What means that all derivations presented so far, and so forth, are applicable to both failure criteria except for a constant value ($\cos\varphi'$).

To end this section, we are going back to the effect of the initial state of stress under which the soil was consolidated. Figure 8, at the beginning of next page, shows the effective stress paths of two specimens of the same tube sample subjected to an undrained axial compression, but each of them having been consolidated differently, although with the same mean effective stress. One of the specimens (CK$_0$UC test) was consolidated under a stress ratio $K = K_0$ that the engineer considered to exist on the field, beneath a large earthfill. However, during the consolidation stage the specimen was allowed to suffer radial strains and contrary to the expectation, this is not but a vulgar undifferentiated CAUC test. Nevertheless, the effective stress path is obviously not the same and the magnitude of excess pore pressures is not the same either. If we simple average everything, as the undrained shear strength measured on these two specimens, we will end up with the average of what cannot meaningfully be averaged.

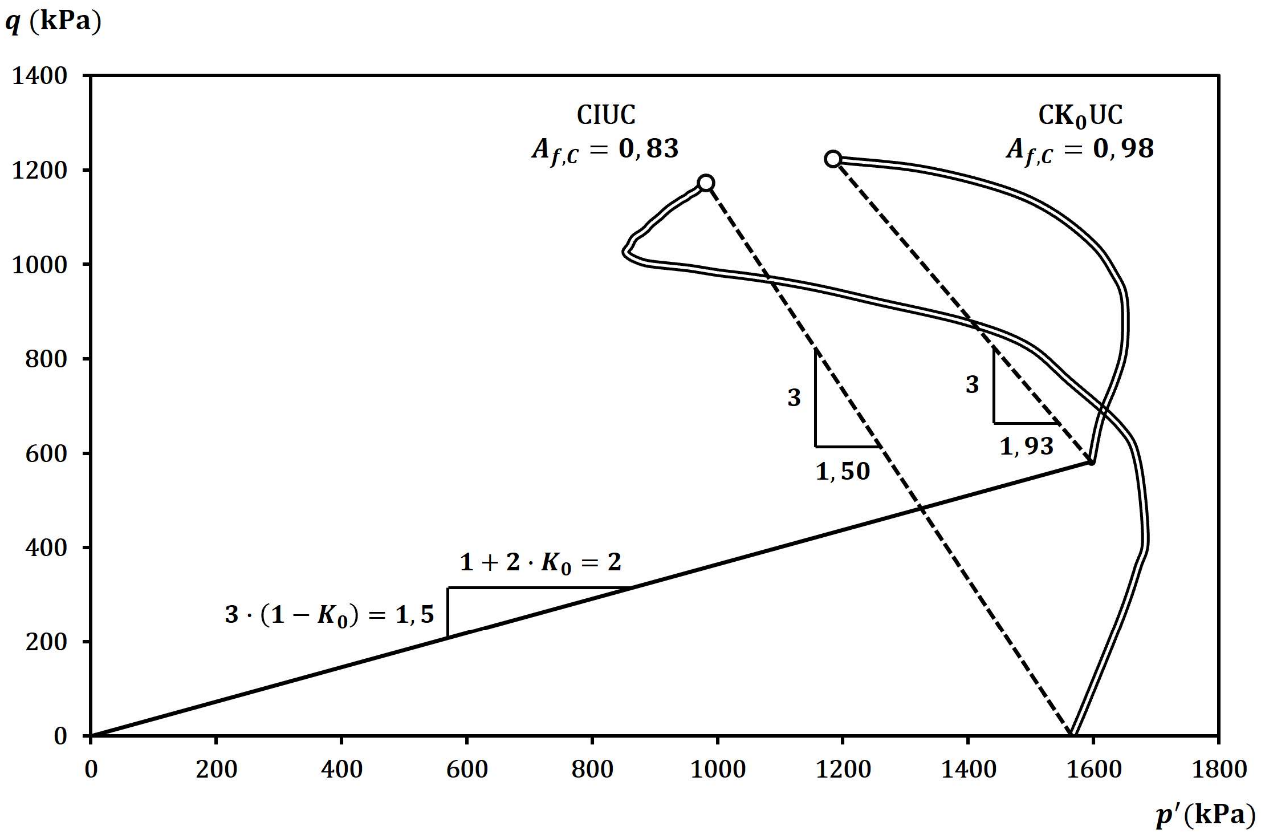

Figure 8

At light of the Critical State theory and according to the Cam Clay model, the pore water coefficient at failure A_f for the general case of a triaxial compression failure of an anisotropically consolidated specimen of normally consolidated soil may be derived. Let Λ be Wroth's plastic volumetric strain ratio and r the spacing ratio of the isotropic consolidation line and the critical state line. As shown on Figure 6, the stress ratio at failure is $M = 6 \cdot \sin\varphi' / (3 - \sin\varphi')$. Also, as shown in figure 8, the stress ratio along the K_0 consolidation line (one dimensional consolidation), call it η_0, is:

$$\eta_0 = \frac{3 \cdot (1 - K_0)}{1 + 2 \cdot K_0} \tag{14}$$

So, the pore water coefficient at failure (or at the critical state) would be predicted from:

$$A_f = \frac{(r \cdot M^2)^\Lambda}{M \cdot ({\eta_0}^2 + M^2)^\Lambda - \eta_0 \cdot (r \cdot M^2)^\Lambda} \times \tag{15}$$

$$\times\left(1-\left(\frac{{\eta_0}^2+M^2}{r\cdot M^2}\right)^\Lambda+\frac{1}{3}\cdot\frac{M\cdot({\eta_0}^2+M^2)^\Lambda-\eta_0\cdot(r\cdot M^2)^\Lambda}{(r\cdot M^2)^\Lambda}\right)$$

If, instead, the specimen had been isotropically consolidated we would have $\eta_0 = 0$ and equation 15 would become:

$$A_f = \frac{1}{M}\cdot\left(r^\Lambda - 1 + \frac{1}{3}\cdot M\right) \qquad [16]$$

If there was a critical state and if we could find it, if M (or φ'_{cs}), r and Λ were intrinsic properties of a soil, then a theoretical interrelationship between the pore pressure coefficients at failure for the cases of isotropically consolidated specimens and one-dimensionally consolidated specimens ($K = K_0$) could established.

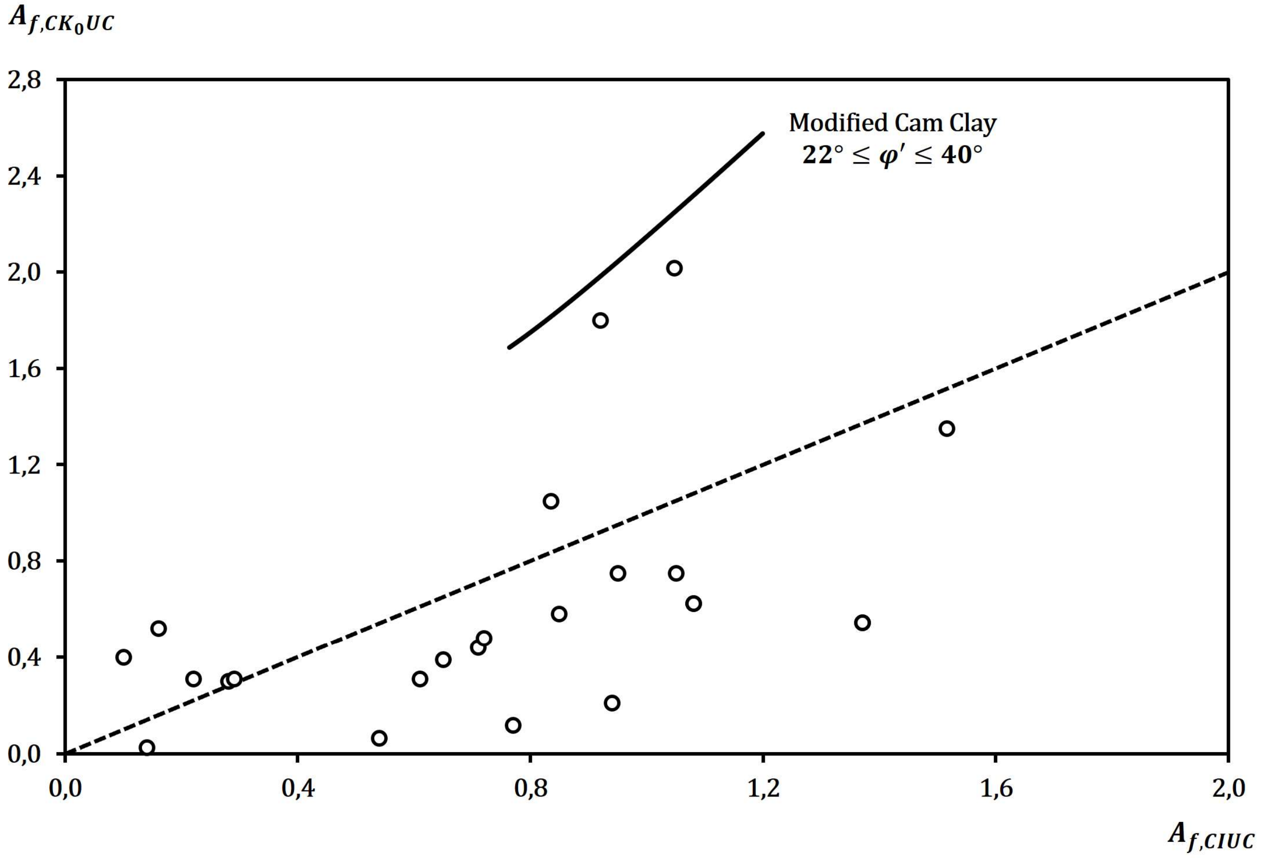

Figure 9

Assuming, as for the Modified Cam Clay, that $r = 2$, assuming also that $\Lambda = 0{,}8$ is a good approximation for most soils and finally assuming that:

$$K_{0,NC} = 1 - \sin\varphi' \qquad [17]$$

It is shown on Figure 9, on the previous page, the predicted interrelationship between the pore pressure coefficients for isotropic (CIUC) and one-dimensional consolidation (CK_0UC) conditions. Simultaneously, 24 pairs of values, each obtained experimentally on specimens of the same soil, are presented and, comparing both, we may conclude that the theoretical model is unable to explain the different generation of excess pore pressures when specimens are differently consolidated and afterwards equally taken to failure by an axial compression. We are using this example to explain that quoting erudite references on geotechnical reports, even if the best available, does not mean that we are closer to reality, nor that anyone else will believe in our faith on theories and models, without evidence consubstantiating them.

3. THE PLANE STRAIN PROBLEM

3.1. A small step towards the "real" field conditions

Unless we jump to into three-dimensionality, if there is something that may resemble the stress-strain conditions of a geotechnical structure like an embankment dam, that will be the plane strain problem, where the lateral out-of-plane strain is null. We may consider remaining on a bi-dimensional problem ahead of jumping to the abyss in what concerns to soil undrained shear strength.

To introduce the problem, we will first write down the expressions of stress on the octahedral plane, the one which unit normal vector makes equal angles with the direction of the three principal stresses. The octahedral mean normal stress σ_{oct} is:

$$\sigma_{oct} = \frac{1}{3} \cdot (\sigma_1 + \sigma_2 + \sigma_3) = \frac{1}{3} \cdot I_1 \qquad [18]$$

Being I_1 the first invariant of the stress tensor. The octahedral shear stress τ_{oct} is:

$$\tau_{oct} = \frac{1}{3} \cdot \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sqrt{\frac{2}{3} \cdot J_2} \qquad [19]$$

Being J_2 the second invariant of the deviatoric component of the stress tensor. Relatively to the invariants used on the previous chapter, it is easily noticed that:

$$p = \frac{1}{3} \cdot I_1 = \sigma_{oct} \qquad [20]$$

And:

$$q = \sqrt{3 \cdot J_2} = \frac{3 \cdot \sqrt{2}}{2} \cdot \tau_{oct} \qquad [21]$$

As a generalisation of Skempton´s pore pressure equation 01, on the late fifties of last century, Henkel proposed a new equation for the pore pressure excesses:

$$\Delta u = \Delta\sigma_{oct} + 3 \cdot a \cdot \Delta\tau_{oct} = \Delta p + \sqrt{2} \cdot a \cdot \Delta q = \Delta u_p + \Delta u_q \qquad [22]$$

Going back to Skempton´s pore pressure equation 1, as a particular case of Henkel's one, we will find that in false triaxiality conditions:

$$\Delta u = \frac{1}{3} \cdot (\Delta\sigma_1 + 2 \cdot \Delta\sigma_3) + a \cdot \sqrt{2} \cdot (\Delta\sigma_1 - \Delta\sigma_3) \Longrightarrow a = \frac{\sqrt{2}}{6} \cdot (3 \cdot A - 1) \qquad [23]$$

These equations will be the basis of the transformation from the laboratory triaxial test conditions to those we know to be the best approximation of the field prevailing ones. Plane strain tests exist but are uncommon in routine geotechnical practice. However, this transformation is indispensable so that the various undrained shear strengths are compatible with the field stress conditions. For the case of false triaxiality, equation 23 just shows that there is an equivalence between Skempton's pore pressure coefficient A and Henckel´s pore pressure coefficient a. What remains to be seen is if there is a match between the values of the pore pressure coefficient a derived from plane strain test measurements and the value obtained by correspondence with triaxial testing.

For the plane strain case, the intermediate effective principal stress, ${\sigma'}_2$, can be expressed by:

$$m = \frac{{\sigma'}_2}{{\sigma'}_1 + {\sigma'}_3} = \frac{(1 - \sin\varphi') \cdot {\sigma'}_1}{{\sigma'}_1 + {\sigma'}_3} \qquad [24]$$

Which presupposes that the out-of-plane principal stress relates to the maximum principal one by Jaky´s equation for the at-rest earth pressure coefficient, already given in equation 17. As the ratio of the maximum and minimum principal stresses at failure is:

$$\frac{{\sigma'}_1}{{\sigma'}_3} = \frac{(1 + \sin\varphi')}{(1 - \sin\varphi')} \qquad [25]$$

We will end up with:

$$m = \frac{{\sigma'}_2}{{\sigma'}_1 + {\sigma'}_3} = \frac{1}{2} \cdot \cos^2\varphi' \qquad [26]$$

Which may also be applied to the respective principal stress increments. Bishop himself, on the sixties of past century, confirmed that equation 26 is a good approximation.

Considering now equation 26, the ratio of the octahedral stresses at failure under triaxial compression (TC) is given by:

$$\left(\frac{\tau_{oct}}{\sigma'_{oct}}\right)_{TC} = \frac{\sqrt{2}\cdot\left(\sigma'_{1f}-\sigma'_{3f}\right)}{\sigma'_{1f}+2\cdot\sigma'_{3f}} = \frac{2\cdot\sqrt{2}\cdot\sin\varphi'}{3-\sin\varphi'} \qquad [27]$$

Considering equations 20 and 21, the slope of the Mohr-Coulomb failure line on the p'- q stress space is:

$$M_{TC} = \left(\frac{dq}{dp'}\right)_{TC} = \frac{3\cdot\sqrt{2}}{2}\cdot\left(\frac{d\tau_{oct}}{d\sigma'_{oct}}\right)_{TC} = \frac{6\cdot\sin\varphi'}{3-\sin\varphi'} \qquad [28]$$

Which was earlier introduced on Figure 6.

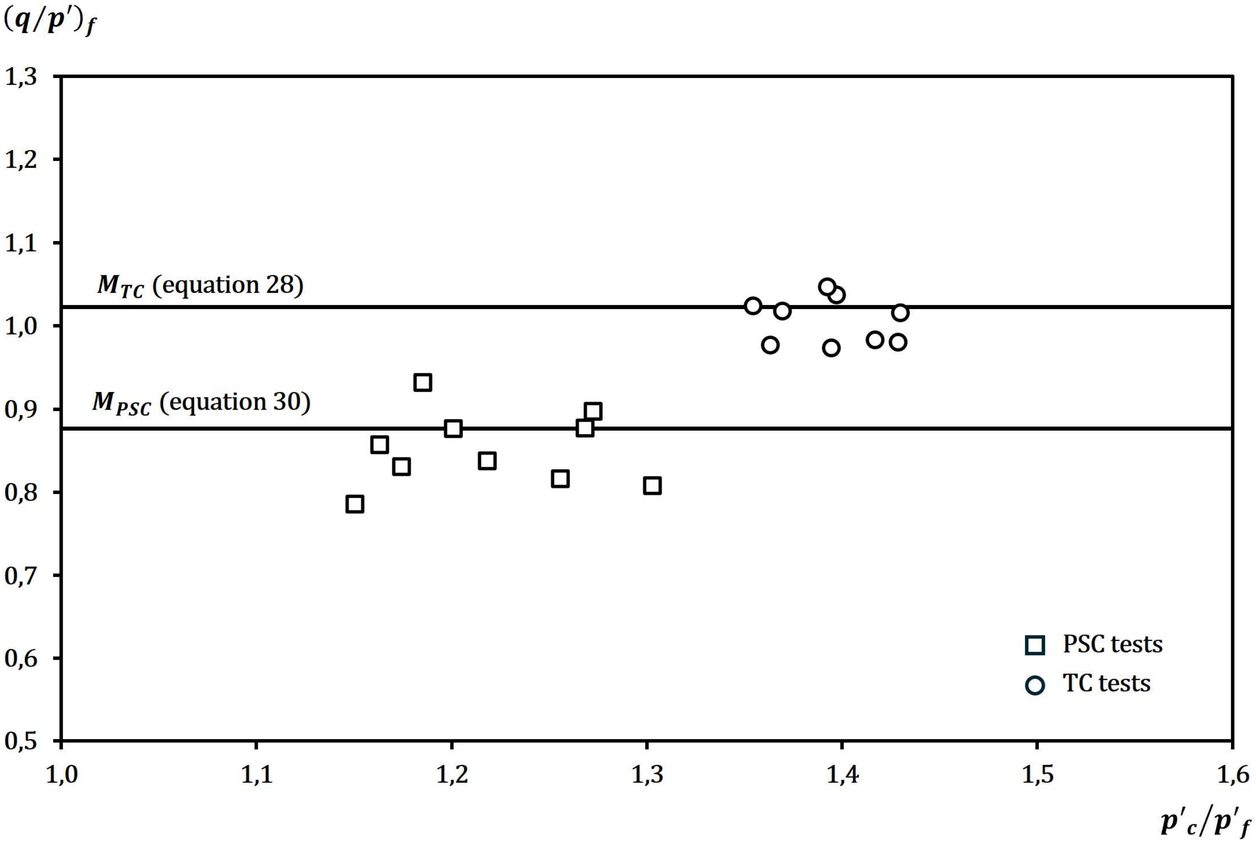

Figure 10

Under a plane strain compression (PSC) failure, the respective octahedral stress ratio is:

$$\left(\frac{\tau_{oct}}{\sigma'_{oct}}\right)_{PSC} = \frac{\sqrt{\left(\sigma'_{1f}-K_0\cdot\sigma'_{1f}\right)^2+\left(K_0\cdot\sigma'_{1f}-\sigma'_{3f}\right)^2+\left(\sigma'_{3f}-\sigma'_{1f}\right)^2}}{(1+K_0)\cdot\sigma'_{1f}+\sigma'_{3f}} =$$

$$= \frac{1}{1+m}\cdot\sqrt{2\cdot m\cdot(1+m)\cdot\left(\sigma'_{1f}+\sigma'_{3f}\right)^2+\frac{1}{2}\cdot\left(1+3\cdot\left(\frac{\sigma'_{1f}-\sigma'_{3f}}{\sigma'_{1f}+\sigma'_{3f}}\right)^2\right)} = \qquad [29]$$

$$= \frac{\sqrt{2} \cdot \sqrt{3 + \sin^2 \varphi'} \cdot \sin \varphi'}{3 - \sin^2 \varphi'}$$

From where the slope of the Mohr-Coulomb failure line on the p'- q stress space is:

$$M_{PSC} = \frac{3 \cdot \sqrt{3 + \sin^2 \varphi'} \cdot \sin \varphi'}{3 - \sin^2 \varphi'} \tag{30}$$

The stress ratios at failure according to equation 28 for an undrained triaxial compression and according to equation 30 for an undrained plane strain compression are plotted on Figure 10, on the previous page, altogether with the results of a series of triaxial and plane strain CK$_0$U tests performed on specimens of the same clay at the Imperial College on the early sixties of last century. On the abscissas of the graph is the mean effective consolidation stress, normalised by the mean effective stress at failure, which is a neutral variable in what regards to the failure criteria.

If a theoretically more consistent and generic way to reach the same conclusions is required, the Mohr-Coulomb failure criteria must be first written as a function of the stress invariants (compression stresses considered positive):

$$-\frac{1}{3} \cdot I_1 \cdot \sin \varphi' + \sqrt{J_2} \cdot \left(\sin\left(\theta + \frac{\pi}{3}\right) + \frac{1}{\sqrt{3}} \cdot \cos\left(\theta + \frac{\pi}{3}\right) \cdot \sin \varphi \right) - c' \cdot \cos \varphi' = 0 \tag{31}$$

Where θ is the Lode angle, given by:

$$\cos(3 \cdot \theta) = -\frac{3 \cdot \sqrt{3}}{2} \cdot \frac{J_3}{{J_2}^{3/2}} \tag{32}$$

And being J_3 the third invariant of the deviatoric component of the stress tensor:

$$J_3 = \left(\sigma'_1 - \frac{1}{3} \cdot I_1\right) \cdot \left(\sigma'_2 - \frac{1}{3} \cdot I_1\right) \cdot \left(\sigma'_3 - \frac{1}{3} \cdot I_1\right) \tag{33}$$

Reintroducing the invariants, p' and q, used previously, equation 31 reads:

$$q = \frac{3 \cdot \sin \varphi'}{\sqrt{3} \cdot \sin\left(\theta + \frac{\pi}{3}\right) + \cos\left(\theta + \frac{\pi}{3}\right) \cdot \sin \varphi} \cdot (p' + c' \cdot \operatorname{cotan} \varphi') \tag{34}$$

From equation 34, it can be concluded that $\theta = \pi/3$ for a triaxial compression failure and that $\theta = 0$ for a triaxial extension failure. Introducing these values of the Lode angle in equation 34, the respective slopes of the failure line on the p'-q stress space will be those indicated on Figure 6 and presented on equation 28 for the triaxial compression case. For the plane strain case we can proceed the same way until equation 30 is obtained.

If equation 26 holds true, it will be concluded that the Lode angle for the plane strain compression is $\theta = 45{,}6°$. The relative value of the intermediate principal stress is also commonly expressed through a parameter b:

$$b = \frac{\sigma'_2 - \sigma'_3}{\sigma'_1 - \sigma'_3} \qquad [35]$$

The respective plane strain value will be $b = 0{,}25$ which lies between a value 0 for triaxial compression and a value of 1 for triaxial extension. The Lode angle can be expressed in terms of parameter b as follows:

$$\cos(3 \cdot \theta) = -\frac{1}{2} \cdot \frac{2 \cdot b^3 - 3 \cdot b^2 - 3 \cdot b + 2}{(b^2 - b + 1)^{3/2}} \qquad [36]$$

In general, from the Lode angle and the two other invariants, p and q, it may be useful to return to the three principal stresses. The pertinent formulae are:

$$\begin{aligned}\sigma'_1 &= \frac{I_1}{3} + 2 \cdot \sqrt{\frac{{I_1}^2}{9} - \frac{I_2}{3}} \cdot \cos\left(\frac{\pi}{3} - \theta\right) = \sigma'_{oct} + \sqrt{2} \cdot \tau_{oct} \cdot \cos\left(\frac{\pi}{3} - \theta\right) = \\ &= p' + \frac{2}{3} \cdot q \cdot \cos\left(\frac{\pi}{3} - \theta\right)\end{aligned} \qquad [37]$$

$$\begin{aligned}\sigma'_2 &= \frac{I_1}{3} - 2 \cdot \sqrt{\frac{{I_1}^2}{9} - \frac{I_2}{3}} \cdot \cos\left(\frac{2 \cdot \pi}{3} - \theta\right) = \sigma'_{oct} - \sqrt{2} \cdot \tau_{oct} \cdot \cos\left(\frac{2 \cdot \pi}{3} - \theta\right) = \\ &= p' + \frac{2}{3} \cdot q \cdot \cos\left(\frac{2 \cdot \pi}{3} - \theta\right)\end{aligned} \qquad [38]$$

$$\sigma'_3 = \frac{I_1}{3} - 2 \cdot \sqrt{\frac{{I_1}^2}{9} - \frac{I_2}{3}} \cdot \cos(-\theta) = \sigma'_{oct} - \sqrt{2} \cdot \tau_{oct} \cdot \cos(-\theta) = p' + \frac{2}{3} \cdot q \cdot \cos(-\theta) \qquad [39]$$

Being I_2 the second invariant of the stress tensor, that is:

$$I_2 = \sigma'_1 \cdot \sigma'_2 + \sigma'_2 \cdot \sigma'_3 + \sigma'_3 \cdot \sigma'_1 \qquad [40]$$

For the plane strain case, making use of equations 24 and 29, the above equations 37 to 39 become:

$$\sigma'_1 = \frac{\sqrt{3} \cdot p'}{2 \cdot (1 + m)} \cdot \left(\sqrt{3} + \sqrt{2 \cdot (1 + m)^2 \cdot \frac{2}{9} \cdot \left(\frac{q}{p'}\right)^2 - (2 \cdot m - 1)^2}\right) \qquad [41]$$

$$\sigma'_2 = \frac{3 \cdot m \cdot p'}{1+m} \qquad [42]$$

$$\sigma'_3 = \frac{\sqrt{3} \cdot p'}{2 \cdot (1+m)} \cdot \left(\sqrt{3} - \sqrt{2 \cdot (1+m)^2 \cdot \frac{2}{9} \cdot \left(\frac{q}{p'}\right)^2 - (2 \cdot m - 1)^2} \right) \qquad [43]$$

Finally, to derive a set of equations for the undrained shear strength under plane strain conditions, a definition of the secant effective stress path to failure is still explicitly remaining. From equation 22, the secant effective stress path to failure on the p'- q stress space is fully defined by Henkel's pore pressure coefficient a:

$$\frac{\Delta p'}{\Delta q} = \frac{\Delta p - \Delta u}{\Delta q} = -\sqrt{2} \cdot a \qquad [44]$$

Which is perfectly general for both the triaxial and the plane strain conditions. It is important to note that we are still admitting that the friction angle is constant, regardless of the loading and strain conditions. Differences, however, can be notable, ascending to 10% from triaxial compression to plane strain compression and ascending to 30% or even more from triaxial compression to plane strain extension. As happened on the context of the previous chapter, incorporating the relative values of the friction angle into the undrained shear strength formulation is straightforward. Neglecting the differences will likely lead to an underestimation of the undrained shear strength for modes of failure other than a triaxial undrained compression.

3.2. The α_1 factor

It has been mentioned earlier that a chain product factor, α_1, would allow to transform the undrained shear strength measured on the reference triaxial test, CIUC type, into one available on the closest conditions to those occurring on field. The first component, α_{11} or α_{12}, allows to transform the reference strength into those mobilised under different consolidation conditions and different loading types, yet still in triaxial stress conditions. Using the principles explained on the previous section, the second component, α_{13} or α_{14}, will allow us, without any loss of generalization, to further transform the triaxial undrained shear strengths into the plane strain condition ones.

Assuming that, even if roughly, Henkel´s pore pressure coefficient a is the same in triaxial and plane strain conditions, for a plane strain undrained compression failure starting from an anisotropic consolidation state (CK$_0$UPSC or simply PSC), we have:

$$\alpha_{13} = \frac{(s_u)_{PSC}}{(s_u)_{CK_0UC}} = \frac{1 + \left(2 \cdot (A_f)_{CK_0UC} - 1\right) \cdot \sin\varphi'}{3 + \left(\frac{\sqrt{3+\sin^2\varphi'} \cdot (3 - \sin\varphi')}{(3 - \sin^2\varphi')} \cdot \left(3 \cdot (A_f)_{PSC} - 1\right) - 1\right) \cdot \sin\varphi'} \times \qquad [45]$$

$$\times \frac{\frac{3}{2} \cdot \frac{\sqrt{3+\sin^2\varphi'} \cdot (3-\sin\varphi')}{(3-\sin^2\varphi')} \cdot \left(K_0 + (1-K_0) \cdot \left(A_f\right)_{PSC}\right)}{K_0 + (1-K_0) \cdot \left(A_f\right)_{CK_0UC}}$$

And, for a plane strain undrained extension failure starting from an anisotropic consolidation state (CK$_0$UPSE or simply PSE) we will have in turn:

$$\alpha_{14} = \frac{(s_u)_{PSE}}{(s_u)_{CK_0UE}} = \frac{1 + \left(2 \cdot \left(A_f\right)_{CK_0UE} - \frac{1}{3}\right) \cdot \sin\varphi'}{3 + \left(\frac{\sqrt{3+\sin^2\varphi'} \cdot (3+\sin\varphi')}{(3-\sin^2\varphi')} \cdot \left(3 \cdot \left(A_f\right)_{PSE} - 1\right) - 1\right) \cdot \sin\varphi'} \times$$

$$\times \frac{\frac{9}{2} \cdot \frac{\sqrt{3+\sin^2\varphi'} \cdot (3+\sin\varphi')}{(3-\sin^2\varphi')} \cdot \left((1+K_0) - \left(2 \cdot \left(A_f\right)_{PSE} - 1/3\right) \cdot (1-K_0)\right)}{(1+K_0) - \left(2 \cdot \left(A_f\right)_{CK_0UE} - 1/3\right) \cdot (1-K_0)} \qquad [46]$$

This way, a good cautious estimate of the CIUC triaxial compression undrained shear strength, measured in laboratory on good quality soil samples, can be transformed into an eventually not so good, but still cautious, estimate of the plane strain compression undrained shear strength:

$$(s_u)_{PSC} = \alpha_{11} \cdot \alpha_{13} \cdot (c_u)_{CIUC} \qquad [47]$$

Or into an estimate of the plane strain extension undrained shear strength:

$$(s_u)_{PSE} = \alpha_{12} \cdot \alpha_{14} \cdot (c_u)_{CIUE} \qquad [48]$$

Notionally:

$$(s_u)_{PS} = \alpha_1 \cdot (s_u)_{CIUC} \qquad [49]$$

However, both plane strain undrained shear strengths defined above apply to what we will be calling ideal loading conditions, whereby the direction of the total principal stresses is known, and failure is attained with a known variation of one of them. The loading that conducts to any of these shear strengths, including on plane strain, can be reproduced experimentally. So, for an isotropic material, we already know how to estimate the upper (PSC) and lower (PSE) bounds of the undrained shear strengths that can be mobilized on field, based on the results of a single standard triaxial laboratory test (CIUC). Ideally, the undrained shear strengths would be derived from experiments, reproducing the expected field conditions, but that is seldom feasible even if dealing with a structure with the importance of a large embankment dam. Alternatively, strong interrelationships between the pore pressure coefficients built from a solid database will allow us to make educated guesses. Educated guesses are eventually better than ignoring what has been so far explained.

3.3. Anistropy induced by loading

Apart from the natural anisotropy of some soils, resulting from their formation process, loading itself causes a directional variation of the undrained shear strength available at each point in the ground. For the case of an embankment dam on a soft soil foundation, principal effective stresses will be progressively rotating inwards towards the toe of the slope, ranging between the ideal case of a plane strain compression failure to the ideal case of a plane strain extension failure. This variation between ideal cases, to some extent reproducible at the laboratory, is the basis to define a generalized yield function covering any intermediate case of principal stress rotation.

A generalized yield surface for the plane strain case may formulated as follows:

$$\frac{s_u}{s_{uC}} = \\ = \sqrt{\left(\frac{\sigma'_{vf} - \sigma'_{hf}}{2 \cdot c_{uC}}\right)^2 + \left(\left(\frac{s_{uC} + s_{uE}}{2 \cdot s_{uC}}\right)^2 - \left(\frac{\sigma'_{vf} - \sigma'_{hf}}{2 \cdot s_{uC}} - \frac{s_{uC} - s_{uE}}{2 \cdot s_{uC}}\right)^2\right) \cdot \left(\frac{4 \cdot s_{uC} \cdot s_{uE}}{(s_{uC} + s_{uE})^2}\right)^{\mathbf{2}}} \qquad [50]$$

Where σ'_{vf} e σ'_{hf} are the effective stresses at failure at each point of the potential sliding surface, on the vertical and horizontal directions respectively. In the real problem, soil at the actual state is not at failure. However, we know the loading that brought a soil element from its past initial state of equilibrium to the actual state, the one relatively to which we want to determine how far away from a generalized failure state the ground is. On equation 50, s_{uC} is the undrained shear strength mobilised under an ideal plane strain compression ($s_{uC} = s_{uPSC}$) and s_{uE} is the undrained shear strength mobilised under an ideal plane strain extension ($s_{uE} = s_{uPSE}$).

For the normal operation of an embankment dam, the situation we are mostly interested on, the self-weight of the embankment followed by the first reservoir filling constitutes the loading responsible for bringing the ground from its initial state to the actual one and we may assume that a radial loading path is a sensible choice to reach a virtual state of failure starting from the actual state. So, we will be imagining a proportional increase of the real loading applied on the past and extend it until soil yielding is attained. Obviously, real new actions, not imagined ones, may force the ground to follow any other load path, starting from the actual state. Those cases are beyond what is conventionally known as the normal operation loading.

Considering then the initial state of stress, described by the effective stresses on the vertical σ'_{v0} and horizontal σ'_{h0} directions and considering the stress increments caused by the loading that brought a soil element from the initial to the actual state, described by the normal stress increments on the vertical $\Delta\sigma'_v$ and horizontal $\Delta\sigma'_h$ directions plus the in-plane increment of tangential stress $\Delta\tau_{vh}$, we may obtain the stresses at failure required in equation 50 by stating:

$$\sqrt{\left(\left(\frac{s_{uC} + s_{uE}}{2 \cdot s_{uC}}\right)^2 - \left(\frac{\sigma'_{vf} - \sigma'_{hf}}{2 \cdot s_{uC}} - \frac{s_{uC} - s_{uE}}{2 \cdot s_{uC}}\right)^2\right) \cdot \left(\frac{4 \cdot s_{uC} \cdot s_{uE}}{(s_{uC} + s_{uE})^2}\right)^{\mathbf{2}}} = \qquad [51]$$

$$= \frac{2 \cdot \Delta\tau_{vh}}{\Delta\sigma'_v - \Delta\sigma'_h} \cdot \left(\frac{\sigma'_{vf} - \sigma'_{hf}}{2 \cdot s_{uC}} - \frac{\sigma'_{v0} - \sigma'_{h0}}{2 \cdot s_{uC}} \right)$$

Which may be simply solved with the quadratic equation for x:

$$x = \left(\frac{\sigma'_{vf} - \sigma'_{hf}}{2 \cdot s_{uC}} \right) = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \qquad [52]$$

With the following coefficients:

$$a = \left(\frac{2 \cdot \Delta\tau_{vh}}{\Delta\sigma'_v - \Delta\sigma'_h} \right)^2 + \left(\frac{4 \cdot s_{uC} \cdot s_{uE}}{(s_{uC} + s_{uE})^2} \right)^{\mathbf{2}} \qquad [53]$$

$$b = -2 \cdot \left(\frac{\sigma'_{v0} - \sigma'_{h0}}{2 \cdot s_{uC}} \right) \cdot \left(\frac{2 \cdot \Delta\tau_{vh}}{\Delta\sigma'_v - \Delta\sigma'_h} \right)^2 - 2 \cdot \left(\frac{s_{uC} + s_{uE}}{2 \cdot s_{uC}} \right) \cdot \left(\frac{4 \cdot s_{uC} \cdot s_{uE}}{(s_{uC} + s_{uE})^2} \right)^{\mathbf{2}} \qquad [54]$$

$$c = \left(\frac{\sigma'_{vf} - \sigma'_{hf}}{2 \cdot s_{uC}} \right)^2 \cdot \left(\frac{2 \cdot \Delta\tau_{vh}}{\Delta\sigma'_v - \Delta\sigma'_h} \right)^2 - 2 \cdot s_{uE} \cdot \left(\frac{4 \cdot s_{uC} \cdot s_{uE}}{(s_{uC} + s_{uE})^2} \right)^{\mathbf{2}} \qquad [55]$$

This way we will reach an estimate of the undrained shearing resistance available under a radial loading in any point of a potential sliding surface. In practice, we may deal with a variable shearing resistance in several ways, either explicitly using a spatial function or using a single smart averaged value. The author is usually satisfied with the latter because of the false sense of exactness of the former.

At this stage, for the reasons we have just discussed, doubts should not exist about the inexistence of a single undrained shear strength along a potential sliding surface, but as many as the stress histories and as many as the loadings that could brought him to failure, whenever assuming that drainage conditions are fictitiously blocked as happened on the laboratory, when the tap was closed on the triaxial apparatus. Undrained shear strength, definitely, is not an intrinsic soil property, despite how much we may enjoy playing with the results of the ground resistance, not soil strength, offered on penetration or pressuremeter field tests. We should not ask anyone for “the” value of the undrained shear strength of a soil, without defining for what we want it to be derived.

To illustrate the application of the above strategy for choosing the undrained shear strength values controlling a failure with blocked drainage, let us suppose, as illustrated in Figure 11 on next page, an embankment dam with a toe drain built over a compressible foundation soil. Ahead of any discussion, we assume having been asked to calculate the downstream slope factor of safety considering a long-term undrained failure of the dam foundation, using a total stress approach. For now, the occurrence of an undrained ground failure is just a supposition that we may refute afterwards. The fact that we know how to do it, does not mean that we agree that needs to be done.

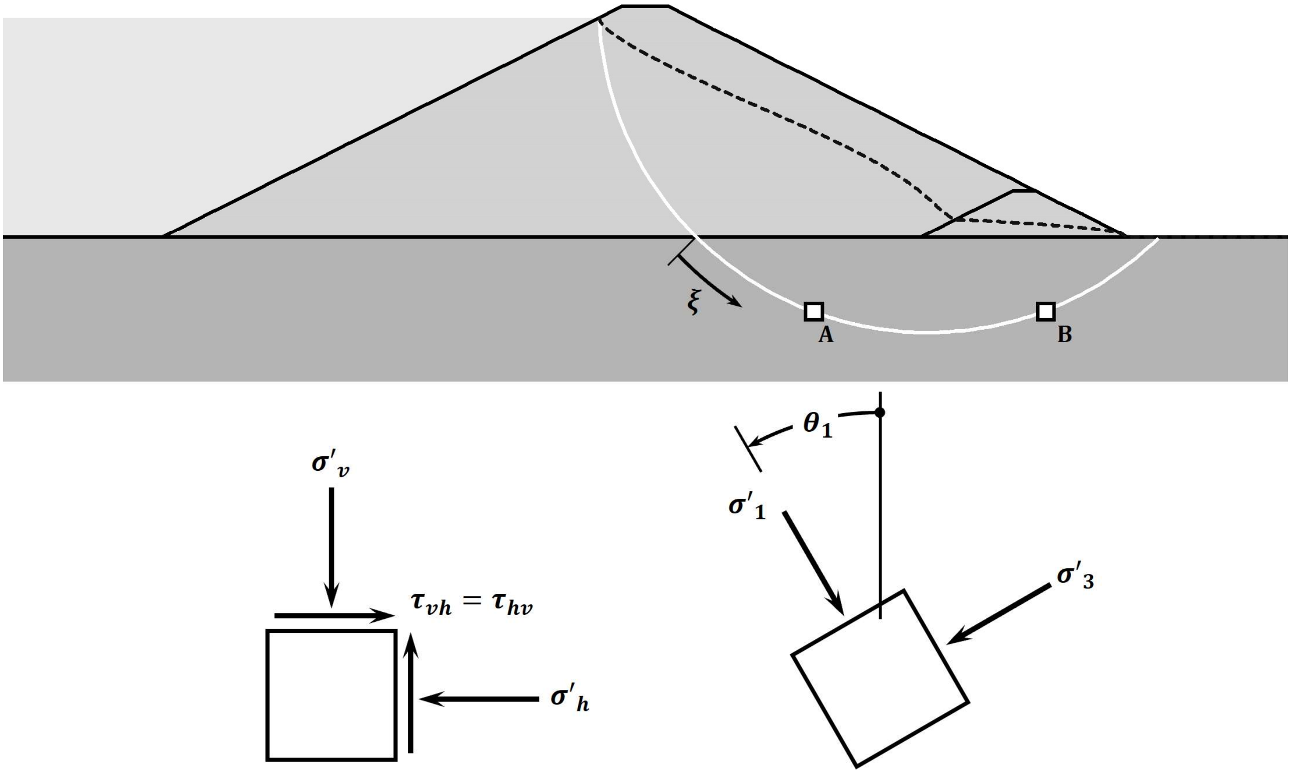

Figure 11

We will suppose that the foundation soil was very slowly loaded by the earthwork's construction, followed by an equally slow first filling. At the actual stage, represented on the figure, there is no undrained shear strength being mobilised (excess pore pressures are null), but we want to determine the downstream slope factor of safety admitting that forcefully the only stress path available towards a virtual failure would demand the full generation of excess pore pressures during shear (conditions of blocked drainage). We will put forward, as an additional simplifying hypothesis, that, at the initial stage, the virgin foundation soil was isotropically consolidated ($K_0 = 1$). Also, we will be admitting that the ideal plane strain extension strength is equal to half of its compressive counterpart.

Soil elements A and B, indicated in Figure 11, are located over a potential failure surface, one fourth of the total slip surface length away from the entrance and exit points on the foundation soil, respectively. On next page, dashed lines in Figure 12 represent the load sequentially applied to bring each soil element from its initial state (points A_0 and B_0) to its actual state (points A_1 and B_1). A radial loading, represented with double line vectors, extends the previous secant loading until it touches the yielding surface of each element (points A_f and B_f), being that equivalent to the application a constant load factor to all the components of the incremental stress tensor. One should note that each element has its own yield surface, that is, each element, depending on the ground stress history, has a different plane strain undrained shear strength under the ideal compression loading, s_{uC}.

The magnitude of the double line vectors in Figure 12 ($\overrightarrow{A_0\,A_f}$ and $\overrightarrow{B_0\,B_f}$) corresponds to the undrained shear strength s_u, normalized by the ideal compressive strength, available at each soil element to resist the imagined radial loading path to failure. The ratio of the distances between the initial and the actual states and the distances between the initial and the yield states is a measure of strength mobilisation at each element, or equivalently, the inverse ratio represents a local factor of safety. Slice methods for limit equilibrium analysis of slope stability can incorporate local factors of safety, which are variable along a postulated sliding surface.

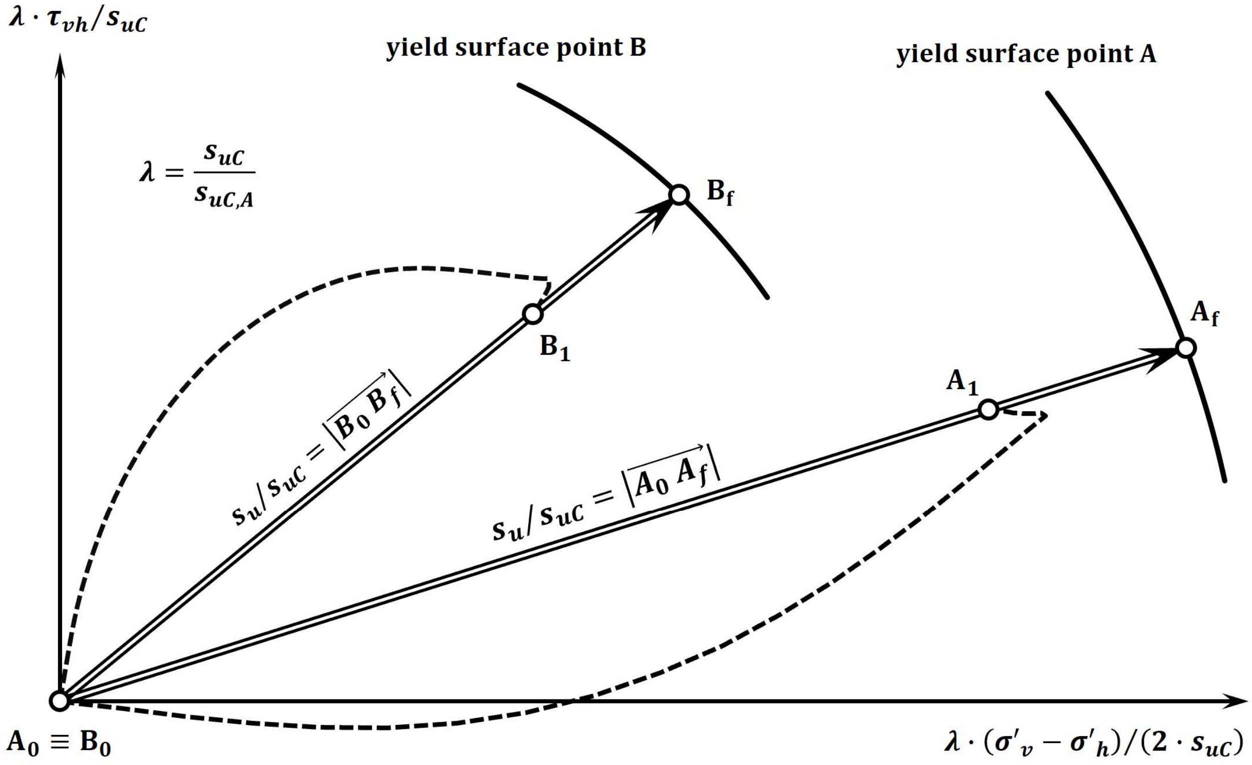
$\lambda \cdot \tau_{vh}/s_{uC}$
yield surface point B
yield surface point A
$\lambda = \frac{s_{uC}}{s_{uC,A}}$
B_f
B_1
A_f
A_1
$s_u/s_{uC} = |\overrightarrow{B_0 B_f}|$
$s_u/s_{uC} = |\overrightarrow{A_0 A_f}|$
$A_0 \equiv B_0$
$\lambda \cdot (\sigma'_v - \sigma'_h)/(2 \cdot s_{uC})$

Figure 12

To illustrate further the strategy, we can show how direct the relationship is between the available shear strength and the rotation of the principal stresses along the potential slip surface shown in Figure 11. In relation to the vertical direction, the maximum principal plane is inclined at an angle θ_1 (positive counterclockwise):

$$\theta_1 = -\frac{1}{2} \cdot \arctan\left(\frac{2 \cdot \tau_{vh}}{\sigma'_v - \sigma'_h}\right) \qquad [56]$$

Figure 13, on next page, shows both the normalized undrained shear strength s_u/s_{uC} and the maximum principal stress rotation θ_1 along the arc length of the slip surface that travels on the dam foundation, given by the curvilinear coordinate ξ measured from the entrance point and normalized by the total arc length L.

A first observation must be emphasized, noting, once again, that the s_{uC} is also variable along the slip surface depending on the stress history of each point. For a normally consolidated soil, an increasing reduction of the plane strain compression strength will take place as we progress towards the toe of the slope (increasing ξ/L). In turn, the over-consolidation effect, discussed in the next section, on the undrained shear strength will, in the opposite sense, increase the plane strain strength.

A second observation lies on the concave shape of the normalised undrained shear strength variation along the slip surface, meaning that, on average, the overall reduction of the available undrained shear strength is, for this type of problem, significantly lower than the average of the ideal compressive and extension normalised strengths (1 for compression and 0,5 for extension, on the example) in plane stain.

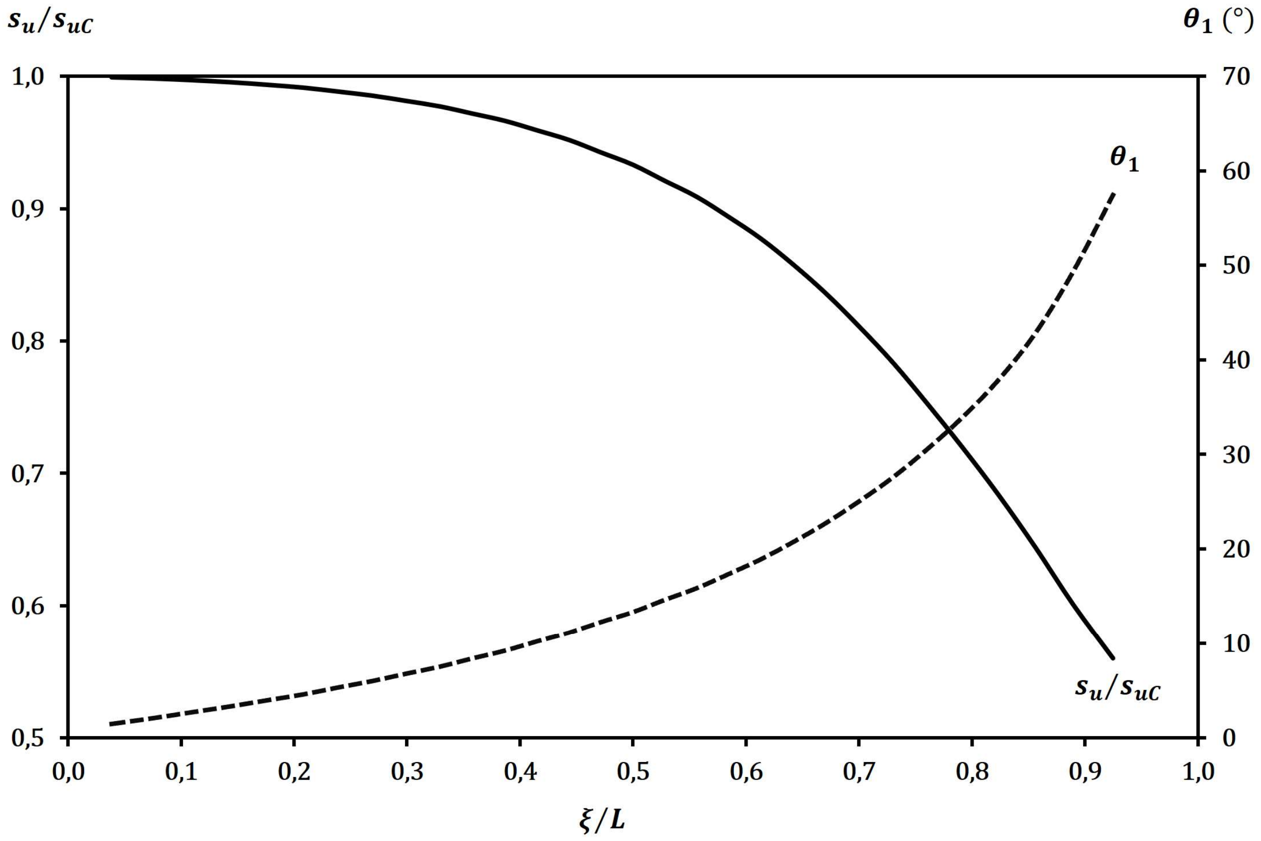

Figure 13

As a rule of thumb, one may expect that averaged shear strength over the slip surface segment crossing the foundation of embankment dam to be:

$$\bar{s}_u = 0{,}75 \times (s_u)_{CIUC} \tag{57}$$

Whichever is the rationale behind the strategy adopted for the selection of an operative value of undrained shear strength for a particular problem, a rationale must exist and needs to be explained beyond the discussion of test results. We remember that the example given in this subchapter corresponds to what is called a long-term undrained failure, under no new external action effect. The same strategy applies to short-term undrained failures, typically occurring during or immediately after a dam embankment construction, when, depending on the ground and the loading rates, undrained failures really occur due to the generation of excess pore pressures. We will get back to this subject, as an argumentative fallacy is intentionally escaping.

So far, in this section, the undrained shear strength available along a postulated slip surface has been estimated based on the stress system, what implies the initial state of stress, and the stress increments caused by a particular loading are known to some extent. We could even admit that the actual state of stress was so far away from failure, that an elastic stress distribution could represent a sufficiently good estimate of that occurring on field.

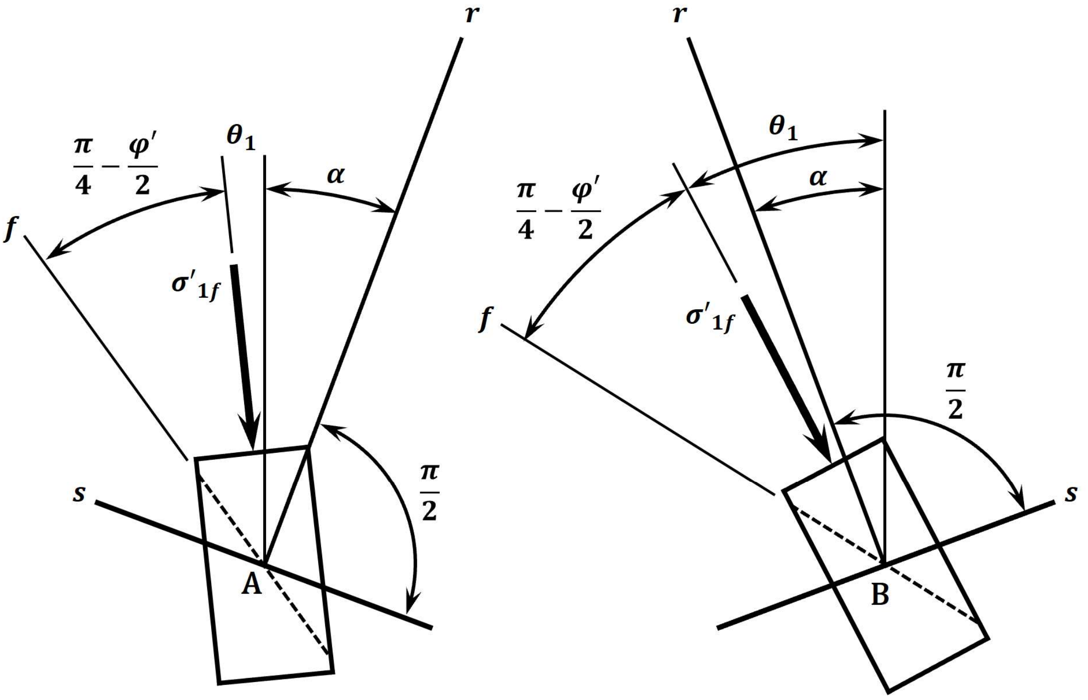

Figure 14

The circular slip surface shown on Figure 11 has been stipulated a priori and completely ignores the distribution of stresses resulting from the embankment construction and subsequent reservoir first filling and therefore its directional effects on the undrained shear strength of the foundation soil. Furthermore, the embankment dam, under its normal operating loading, is not supposed to be on the imminence of collapse. For the two soil elements, A and B, located along the slip surface as indicated on Figure 11, the presumable actual direction, θ_1, of the actual maximum principal effective stress, σ'_1, is represented on Figure 14. The radius of the slip surface is represented by line r and the direction of sliding is given by the tangent lines s to the slip surface, at each point, forming an angle α with the horizontal direction.

If the principal stresses were virtually increased, without further rotation, until failure occurred at each element, the planes of failure, represented by dashed lines f, would form an angle of $\pi/4-\varphi'/2$ with the direction of the maximum principal stress. Hence, the slip surface is not orientated coincidently with the planes of soil failure that would develop if a proportional load increment was imagined, as illustrated on Figure 12. However, there are no stress increments remaining to be applied on the situation under analysis, so talking about further stress rotation is an abstraction. The most logical conclusion is that the failure mechanism arbitrarily admitted and drawn on Figure 11 is not representative of the loading situation under analysis. In fact, the slip surface on Figure 11 is the worst (lowest factor of safety) of those that can be considered representative of an earth dam catastrophic failure and was obtained with an arbitrary constant value of undrained shear strength.

The strategy described before to estimate the effect of the stress induced anisotropy of undrained shear strength will tend to push the critical slip surface outwards, in such a way that lower undrained strengths are mobilised on the passive side, that is, when the slip surface is directed upwards.

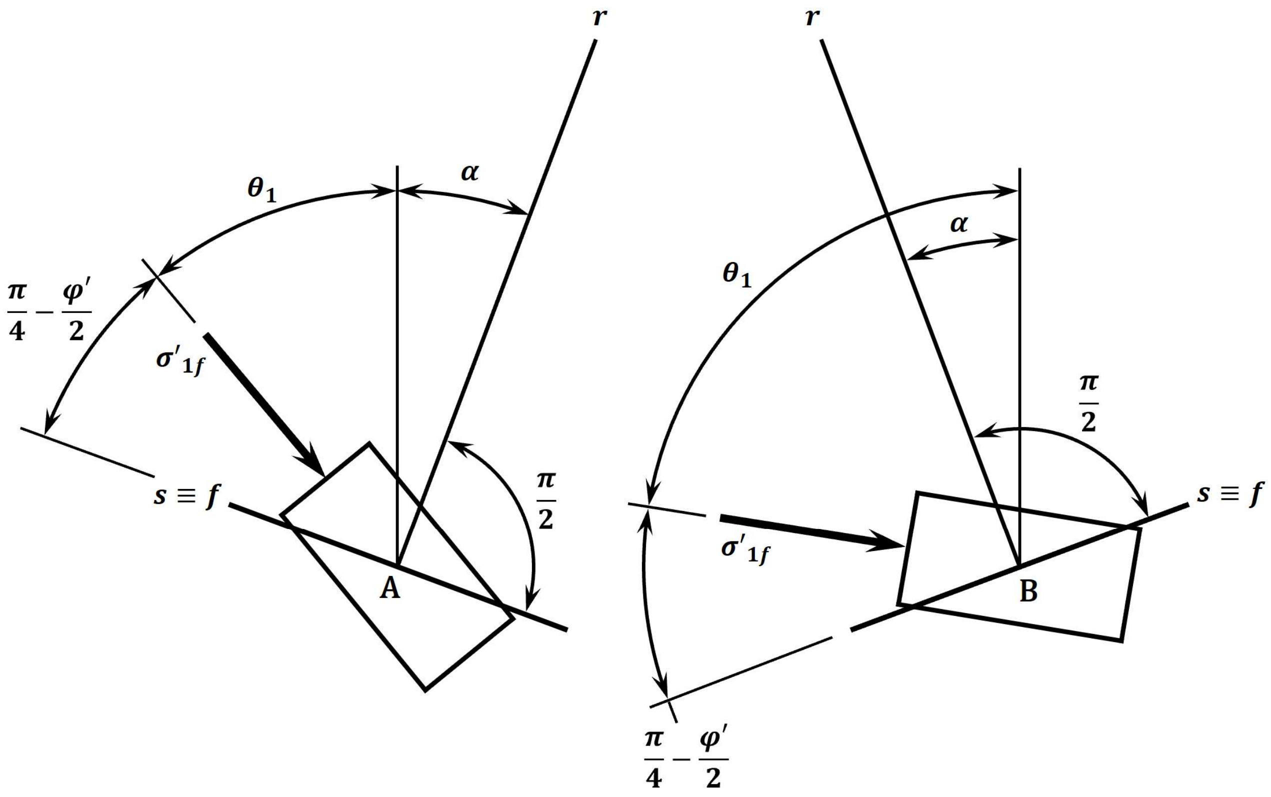

Figure 15

An alternative approach, a popular one indeed, uses the punctual direction of an arbitrary slip surface to define the direction of the maximum principal stress, as represented on Figure 15 for the same two soil elements, A and B, indicated on Figure 11:

$$\theta_1 = \frac{\pi}{4} \mp \alpha + \frac{\varphi'}{2} \qquad [58]$$

In such a way that the ratio s_u/s_{uC} can be expressed irrespectively of the stress system present of the soil, meaning that no knowledge of the actual stresses ($\sigma'_{v0} + \Delta\sigma_v$, $\sigma'_{h0} + \Delta\sigma_h$ and $\tau_{vh0} + \Delta\tau_{vh}$) would be required for a slope stability analysis by the method of slices. So, whichever the location of a postulated sliding surface in relation to the load that would produce it, the direction of the maximum principal stress is imaginarily forced to rotate so that the planes of failure coincide with the sliding surface, $s \equiv f$. In other words, the same value of undrained shear strength would be used in any position of space, as long as the direction of the postulated slip surface at a particular position formed an arbitrary angle α with the horizontal direction. That seems unreasonable.

Nevertheless, following this alternative approach, the increments of principal effective stress required to bring an element of soil from its actual state to failure would be:

$$\Delta\sigma'_1 = \frac{s_u(\theta_1)}{\sin\varphi'} - \frac{1+K_0}{2}\cdot\sigma'_{v0} + \\ + \sqrt{s_u(\theta_1)^2 - (1-K_0)\cdot\sigma'_{v0}\cdot s_u(\theta_1)\cdot\cos(2\cdot\theta_1) + \left(\frac{1-K_0}{2}\cdot\sigma'_{v0}\right)^2} \quad [59]$$

$$\Delta\sigma'_2 = m\cdot(\Delta\sigma'_1 + \Delta\sigma'_3) \quad [60]$$

$$\Delta\sigma'_3 = \frac{s_u(\theta_1)}{\sin\varphi'} - \frac{1+K_0}{2}\cdot\sigma'_{v0} - \\ - \sqrt{s_u(\theta_1)^2 - (1-K_0)\cdot\sigma'_{v0}\cdot s_u(\theta_1)\cdot\cos(2\cdot\theta_1) + \left(\frac{1-K_0}{2}\cdot\sigma'_{v0}\right)^2} \quad [61]$$

Where $s_u(\theta_1)$ denotes the directional dependent undrained shear strength. From equation 22 we have:

$$\Delta p' + \sqrt{2}\cdot a\cdot\Delta q = 0 \quad [62]$$

Using equations 59 to 61 to determine the increments of stress invariants $\Delta p'$ and Δq and introducing them on equation 62 leads to a quadratic equation in terms of $s_u(\theta_1)$:

$$\frac{\lambda - \sin^2\varphi'}{\sin^2\varphi'}\cdot s_u(\theta_1)^2 + \big((1-K_0)\cdot\sin\varphi'\cdot\cos(2\cdot\theta_1) - \lambda\cdot(1+K_0)\big)\cdot\frac{\sigma'_{v0}}{\sin\varphi'}\cdot s_u(\theta_1) + \\ + \frac{\lambda\cdot(1+K_0)^2 - (1-K_0)^2}{4}\cdot{\sigma'_{v0}}^2 = 0 \quad [63]$$

Being:

$$\lambda = \frac{2\cdot(1+m)^2 - 9\cdot\big(1 - 4\cdot m\cdot(1-m)\big)\cdot a^2}{27\cdot a^2} \quad [64]$$

We can observe on equation 63 that the directional dependency of the undrained shear strength would disappear if the soil element had been isotropically consolidated ($K_0 = 1$), what does not happen on the previous approach based on the presumable stress increments caused by loading. This latter approach was first proposed by Hansen and Gibson at the end of the forties of last century, soon after Skempton had presented his first version of the pore pressure equation. Here, Hansen and Gibson’s equation has been updated using Henkel´s pore pressure equation for the plane strain problem. Duncan and Seed, in the middle of the sixties of last century, updated the same equation using Skempton´s final version of his pore pressure equation. Interestingly, these last authors, included an anisotropic pore pressure coefficient, that could be also updated with Henkel’s equations, as follows:

$$a(\theta_1) = a(\theta_1 = \pi/2) + \big(a(\theta_1 = 0) - a(\theta_1 = \pi/2)\big) \cdot \sin^2 \theta_1 \qquad [65]$$

Where $\theta_1 = \pi/2$ corresponds to a plane strain extension failure and $\theta_1 = 0$ corresponds to a plane strain compression failure, which are the ideal cases reproducible experimentally.

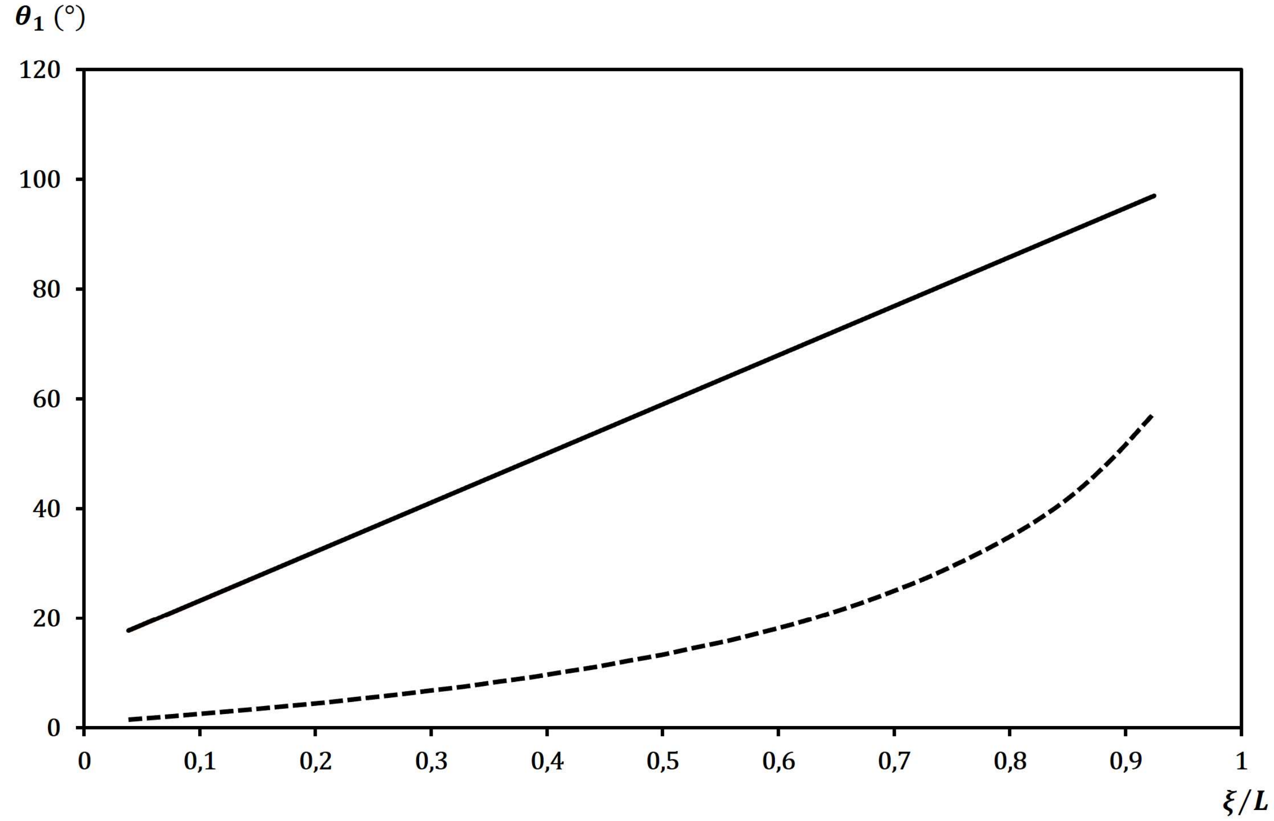

Figure 16

For the slip surface shown on Figure 11, the variation with the curvilinear coordinate, ξ/L, of the angle formed between the maximum principal stress direction and the vertical direction, θ_1, would be as indicated on Figure 16 with a continuous line, if equation 58 was used to determine the direction of the maximum principal effective stress at failure. The dashed line is the same previously presented on Figure 13 and was calculated using the actual increments of stress generated by the dam loading. For this alternative approach to be valid, a brutal rotation of principal stresses would have to take place for each soil element to be brought to failure from its actual state, while the virtual load increase was not causing any rotation. As, on our example, the soil had been isotropically consolidated, the undrained shear strength would be constant along the slip surface and equal to its maximum value, s_{uc}. Exactly the contrary of what we tried to explain.

3.4. Effect of stress history - The α_2 factor

If we admit, even if just approximately, that the effect of the soil stress history on the undrained shear strength is the same regardless of the failure mode, an independent component, call it the α_2 factor, can be added to the chain product used formerly to transform the undrained shear strength measured on the reference test CIUC to that expected on the field. The α_1 factor will apply to normally consolidated (NC) soils and the α_2 factor will incorporate over it the effect of over-consolidation (OC). It is not often that a normally consolidated soil is found in the real world and even a slight over-consolidation increases the undrained shear strength to a point that cannot be disregarded. Notionally:

$$(s_u)_{OC} = \alpha_2 \cdot (s_u)_{NC} \tag{66}$$

Meaning that from the CIUC shear strength of a normally consolidated soil derived from testing, an estimate of the available undrained shear strength on field can be obtained, also including on it the effect of the soil stress history:

$$s_u = \alpha_2 \cdot \alpha_1 \cdot (s_u)_{CIUC,NC} \tag{67}$$

The over-consolidation ratio R may be defined as:

$$R = p'_{max}/p' \geq 1 \tag{68}$$

Where p' is the mean normal stress on the actual state and p'_{max} is the maximum mean normal stress to which the soil has been subjected on the past. Obviously, for a normally consolidated soil $R = 1$. Assuming that soil response can be normalized relatively to the mean consolidation stress, the factor α_2 can be expressed on a simple exponential form:

$$\alpha_2 = R^{\Lambda} \tag{69}$$

Where the exponent Λ can be interpreted as being Wroth's plastic volumetric strain ratio, mentioned earlier on Chapter 2. Over-consolidation may be the simultaneous result of several geological processes, and some may be reproduced by further elaborating p'_{max} on equation 68 as a function of the known intervening factors. An example is the effect of ice overburden stress removal on the undrained shear strength of glacial clays. For other soils, like tropical residual soils, over-consolidation just can be phenomenologically described by equation 68.

A good example to be given here applies to the stress history of the foundation soil underneath the embankment dam represented in Figure 11. Before the slow reservoir first filling, the mean normal effective stress at a soil element on the foundation was p'_1 and the pore water pressure, still free of the reservoir action, was u_1. Some years later, the pore water pressures slowly increased to $u_2 > u_1$ and stabilized under the permanent seepage regime corresponding to the full reservoir level. The effective mean stress was reduced by an amount of $u_1 - u_2$ and become $p'_2 < p'_1$. However, the undrained shear strength did not reduce as much:

$$\frac{s_{u2}}{s_{u1}} = \left(\frac{p'_1}{p'_2}\right)^{\Lambda} = \left(\frac{p'_1}{p'_1 + (u_1 - u_2)}\right)^{\Lambda} \tag{70}$$

Tu put some figures, suppose that the mean effective stress decreased by 33% due to the water table rise. In such case, taking $\Lambda = 0{,}8$, the undrained shear strength would just have dropped less than 8%. One may consider this example to be childish, but not everyone may have seen on dam safety audits what the author repeatedly did. History matters and not just on the mechanics of soils.

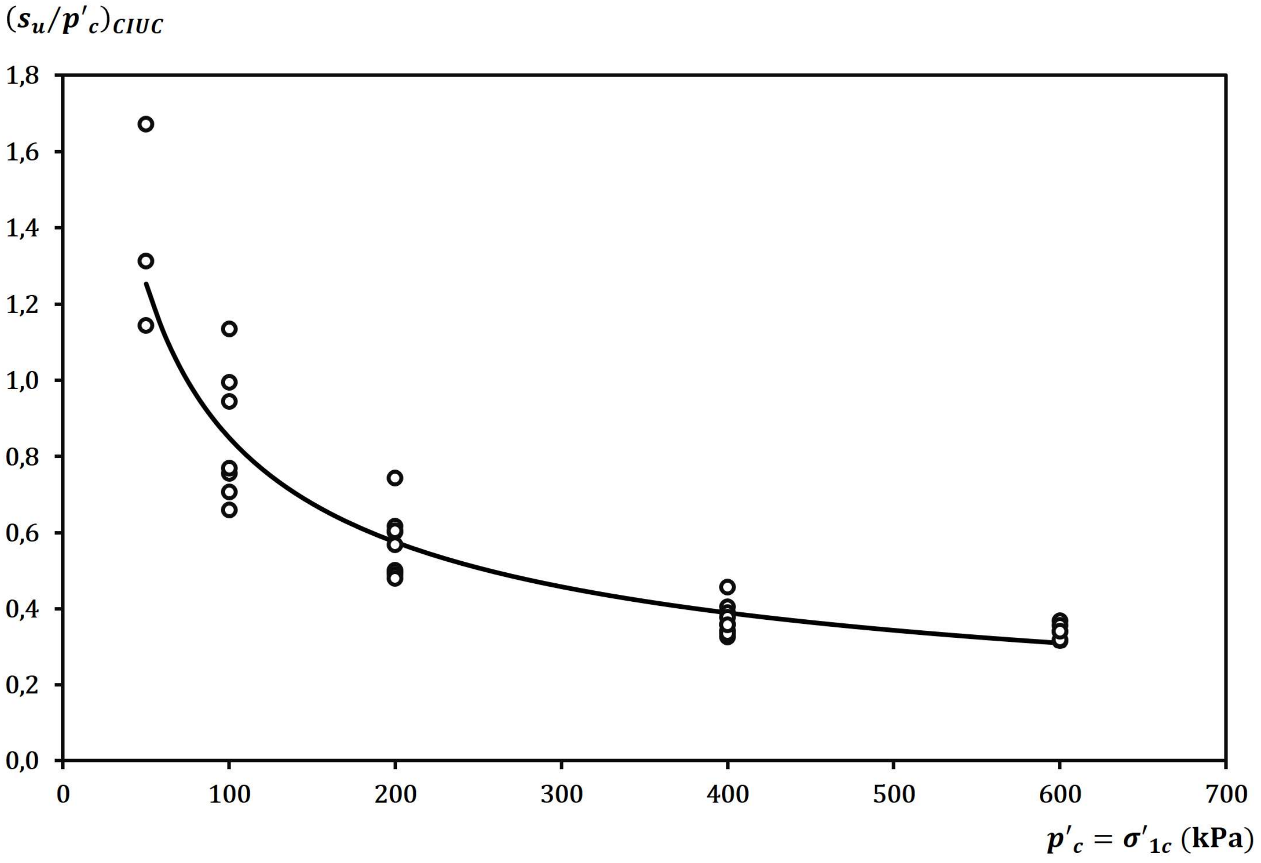

Figure 17

Figure 17 shows the results of 11 triaxial CIUC tests performed on a total of 34 specimens prepared from tube samples of a low plasticity clayey alluvium of medium consistency found on the foundation of an embankment dam, nearly half a century after its construction. The line on the graph is simply a trendline of characteristic values, that is an estimate of the mean shear strength expected on the whole population, based on the mean of the available sampling. At any point of the dam foundation, where the alluvium was detected, the mean normal effective stress is as high as the maximum value of the stress used to isotopically consolidate the specimens, $p'_c = 600$kPa.

As it is, apart from suggesting that the normally consolidated state corresponds to a mean stress around the maximum used to consolidate the specimens, Figure 13 just tells us that all the tested specimens consolidated to lower stresses were over-consolidated. Tests should have been performed on specimens consolidated above the presumable maximum past mean stress. It is not the over-consolidation of a specimen that we seek to know, rather the over-consolidation state on the field. If the samples had all been taken close to surface from a levelled ground, we could expect the degree of over-consolidation on site to be like that revealed by the specimens, even if disregarding the sampling effects. The trendline shown on the figure could even be considered for geotechnical calculations. But this is not the case, as the samples were retrieved from beneath the downstream slope of the dam, and each was subjected to different in-situ stress levels.

On the absence of any other information, an estimate of the normally consolidated undrained strength $(s_u)_{CIUC,NC}$ and an estimate of the mean normal stress at the sampling locations, p'_{field}, is sufficient to derive a profile of over-consolidation on field. Selecting first the undrained shear strength $(s_u)_{OC}$ measured on a specimen consolidated to the same known in-situ mean effective stress ($p'_{test} = p'_{field}$), the specimen over-consolidation ratio is obtained:

$$R_{test} = \left(\frac{(s_u)_{OC}}{(s_u)_{NC}}\right)^{1/\Lambda} \quad [71]$$

From this ratio, the in-situ maximum mean effective stress on field would be:

$$p'_{max,field} = R_{test} \cdot p'_{field} \quad [72]$$

And finally, the in-situ over-consolidation ratio could be estimated:

$$R_{field} = \frac{p'_{max,field}}{p'_{field}} \quad [73]$$

If the above simple steps are not earlier taken into consideration while prescribing the laboratory tests, it is likely that the testing program fails to provide what is required. Complementary tests, providing a description of the soil compressibility, will enhance our capability to investigate the effect of the stress history on the undrained shear strength of a soil.

At the beginning of this chapter, the independence of transformation factors, α_1 and α_2, has been admitted, meaning that the effect of stress history traduced by the latter could be multiplicative of the former, regardless of the stress system acting on a soil element. It is known that not to be true. However, noting that the dependency between both factors must lie on the exponent Λ on equation 69, equation 67 does not lose its conceptual applicability as nothing precludes the use of different exponents, as adequate to the stress system. Typically, for a plane strain compression the value of the exponent Λ will be in the order of 0,73, whereas for a plane strain extension the value will be in the order of 0,82.

4. UNDRAINED FAILURES

4.1. Point of order

On Chapter 2, the different undrained shear strengths of a soil specimen measured on the triaxial apparatus were addressed. On Chapter 3, the transformation of the undrained shear strength measured on a reference standard test into one closer to that available at field on each soil element has been attempted. Some other aspects of great interest have been left aside, namely the effects of soil sampling or the rate of undrained shearing. It has been argued for long that all the individual contributing effects to soil undrained shear strength have a counterbalancing result ($\sum \alpha_i = 1$), annulling each other. If that was so, little merit would have any attempt to quantify only some of the effects, leaving others without consideration. One may conclude that, depending on the effort placed to contemplate all the intervening effects on the estimation of undrained shear strengths, analysis of undrained failures can yield very different results for the same problem.

On both previous chapters, we have been specifically looking into the failure state of a soil element, forced, either on the laboratory or on the field, by the intentional application of a loading increment. On the background, we have admitted that, either on the laboratory or on the field, drainage is blocked, resulting on failures at constant volume. Believing that if something can explain soil shearing resistance, that is the interparticle friction, traduced by a friction coefficient ($\mu = \tan \varphi'$), then an alternative explanation, mixing pore pressures generated by loading on the definition of strength, is just a useful expedient, valid for a particular soil element under a particular loading. If a state of undrained failure was to be analysed, either in the laboratory or on the field, there would not be any dust covering our library of beliefs. For the analysis of real failures of embankments on soft soils, an effective stress approach, including the excess pore pressures generated by the ultimate loading, would be in principle completely consistent with theory and equivalent to a total stress analysis. Soil strength would still be represented by the friction coefficient and the excess pore pressures would be a component of the stress changes caused by the ultimate loading according to the principle of effective stress.

But, for the case that motivates this writing, we are supposedly not dealing with a failure, rather trying to ascertain if there is some distance between the actual state of stress resulting from loading and the state of failure, that we admit being sufficiently well described by the Mohr-Coulomb criteria in terms of effective stresses. Should we consider the excess pore pressures generated by the actual loading, eventually none, or should we consider the excess pore pressures generated by the ultimate loading in particular conditions of blocked drainage, even if fictive on the field? This is an old dilemma that keeps coming back and few seem to be willing to profess a true belief. Wiping the dust slowly, we will leave to the very end the swirling storm that hit us. For now, we will try to discuss the concept of safety and some of its alternative definitions.

4.2. Strength mobilisation

We have seen that there are infinite effective stress paths (ESP) to take a soil element from its actual state (index 0) to a state of failure (index f). We have also seen that a secant effective stress path is uniquely defined by a single pore water pressure coefficient, A_f, for axisymmetric loading conditions, that traduces the excess pore pressures generated by a deviatoric stress increment.

Following the Mohr-Coulomb failure criterion, strength mobilization or its inverse, the factor of safety FS, can be faced as the ratio between the shear stress mobilised at failure τ_f and the shear stess τ, acting on the same plane of failure, as required to ensure a certain state of equilibrium, that we will reckon to be the actual one (index 0). For an anistropically consolidated specimen subjected to a compressive loading in conditions of false triaxiality with blocked drainage, the factor of safety would be:

$$FS = \frac{\tau_f}{\tau} = \frac{2 \cdot \left(\sigma'_{30} + A_f \cdot (\sigma'_{10} - \sigma'_{30})\right)}{\left(\operatorname{cosec} \varphi' - 1 + 2 \cdot A_f\right) \cdot (\sigma'_{10} - \sigma'_{30})} \qquad [74]$$

Where σ'_{10} and σ'_{30} are, respectively, the maximum and minimum principal effective stresses for the actual state of stress and σ'_{1f} and σ'_{3f} ditto for the state of failure. It should be noted that a null value of the pore pressure coefficient A_f does not imply that equation 74 corresponds to the factor of safety in conditions of free drainage ($\Delta u = 0$), unless it coincidentally happens that the ratio between the minimum and maximum increments of principal total stress applied after the consolidation stage fulfill the following condition:

$$\frac{\Delta\sigma_3}{\Delta\sigma_1} = \frac{A_f}{A_f - 1} \qquad [75]$$

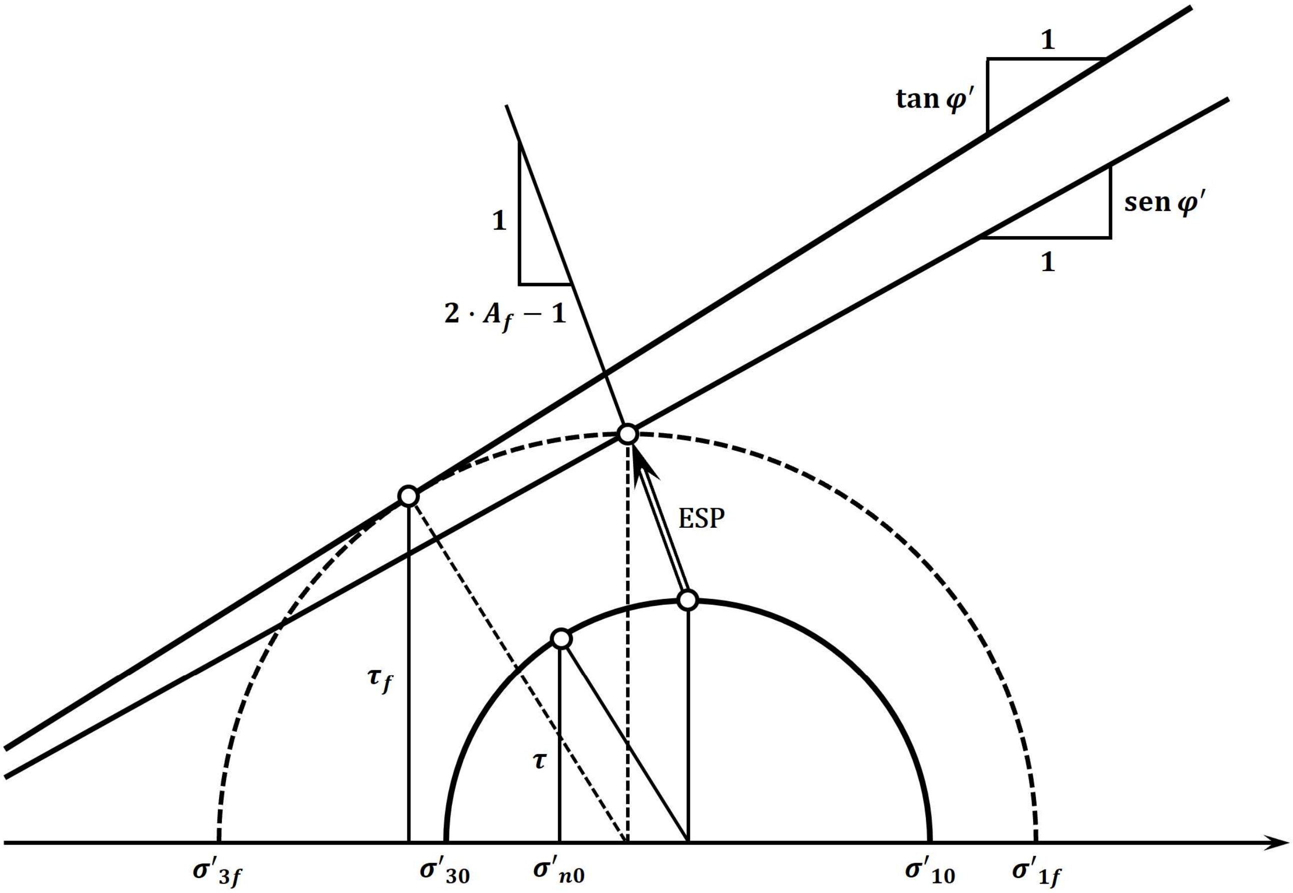

Figure 18

Figure 18 schematically represents the equated problem in its more general form. If drainage conditions are blocked, as on the laboratory by “closing the tap”, the soil element will suffer an undrained failure, whichever is the pore pressure coefficient that describes the loading path. Excess pore pressures, with blocked drainage conditions imposed, may result positive, null or negative. If null, the undrained shear strength mobilised in undrained conditions will be casually the same as the shear strength mobilised under fully drained conditions. If negative, the undrained shear strength mobilised in undrained conditions will be greater than the shear strength mobilised under fully drained conditions, and it is arguable that the study of this case is of any practical interest.

As our focus is on the slope stability of embankment dams it is worth a quick jump to the limit equilibrium analysis by the method of slices. On its most basis assumption, it is considered that the normal stress acting of the putative plane of failure is the same at the actual state σ'_{n0} and at the state of failure σ'_{nf}. Under such assumption, the factor of safety of an element subjected to an axisymmetric loading is:

$$FS = \frac{\tau_f}{\tau} = \frac{\left((\sigma'_{10} + \sigma'_{30}) - (\sigma'_{10} - \sigma'_{30}) \cdot \operatorname{sen} \varphi'\right) \cdot \tan \varphi'}{(\sigma'_{10} - \sigma'_{30}) \cdot \cos \varphi'} \qquad [76]$$

Which presupposes a certain, among many possible, secant effective stress path to failure to be followed. Comparison of equations 74 and 76 is very informative. Both equations yield the same result on two conditions. One condition is:

$$FS = 1 \qquad [77]$$

And the other one is:

$$A_f = \frac{1}{2} \cdot (1 - \operatorname{sen} \varphi') \qquad [78]$$

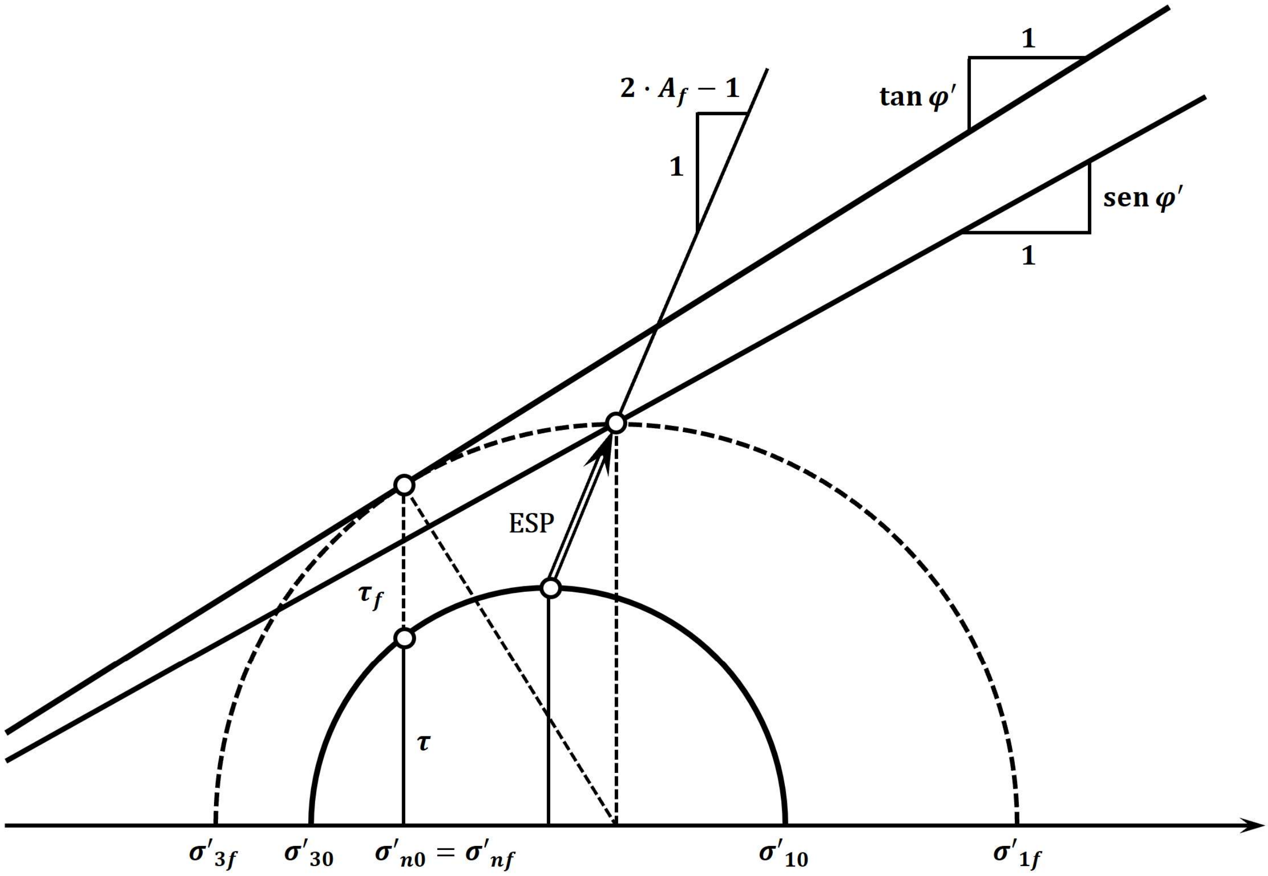

Figure 19

The first condition demystifies the idea that the prospect of an undrained failure may be missed if just drained analyses are performed. If an undrained analyses conducts to a factor of safety close to unity, the same should happen with the corresponding drained analysis. If that is not the case, lack of correspondence may exist between the strength parameters used on both analyses. Also, it is good to remember that factors of safety cannot be lower than unity, as unsafety is not measurable. The second condition reveals that the so-called drained analyses are not as drained as it would seem. From a practical point of view, this second condition gives us a hint about the value of the pore water coefficient within which undrained analyses will provide greater factors of safety than the drained ones and the former became excusable. Figure 19, on the previous page, schematically represents the problem as formulated on equation 76.

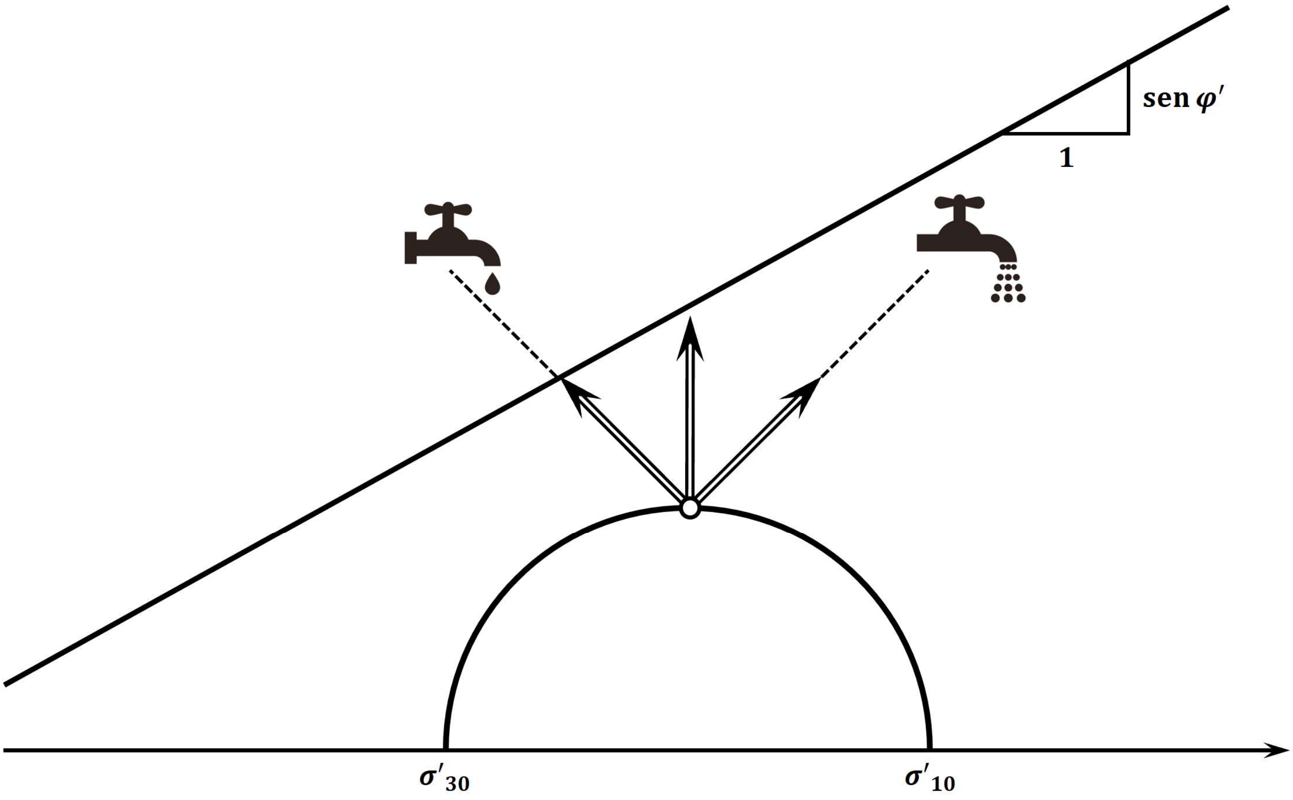
$\mathbf{sen}\,\boldsymbol{\varphi'}$
1
$\boldsymbol{\sigma'}_{30}$
$\boldsymbol{\sigma'}_{10}$

Figure 20

Figure 18 and Figure 19 may be misleading as the Mohr circles representing the state of failure may suggest that, like on a laboratory test, on a real problem there is effectively a load increment forcing the soil element to fail, while, on this chapter, we are simply interested to define how far from a failure state the soil is on its actual state and there are infinite paths available to be followed for a failure to happen. We are concerned with the definition of a margin of safety. It is better to go straight back to the beginning and better look at Figure 20, where the range of available secant effective stress paths to failure is schematically shown, but just for the simplest case of a compression failure. How many distances can be measured between a point and a line?

For each stress path a different shear strength corresponds. On a real problem, that not a laboratory experiment, there are no taps to be closed. On a real problem, that is not a laboratory experiment, a load increment to force a failure may even not even exist, as is the case we are mostly interested in discussing here. The problem we are facing is that of a choice. Which stress path should we choose to define a factor of safety and to which loading conditions should it be applied? Even better, why do we really need to choose a stress path to define a condition of safety? Do we really need to have a single-value measure?

4.3. Reliability and factor of safety

It may be interesting to distinguish the factor of safety from a load factor as to some extent it may seem to be faced as in the previous subchapter. Let us consider that either the soil resistance R and the loading (action or action effect) S are random variables, mutually independent and the occurrence of each follows its own normal probability distribution, with the respective mean value and standard deviation, (μ_R, σ_R) and (μ_S, σ_S) respectively. A central factor of safety would simply relate the expected values of R and S:

$$FS_c = \frac{\mu_R}{\mu_S} \qquad [79]$$

Here, we believe that no one has ever confused the mean value of sampling with the expected value of the population from where the samples were taken. Nevertheless, to make such distinction, a nominal factor of safety may as well be defined:

$$FS_n = \frac{R_k}{S_k} = \frac{\mu_R - k_R \cdot \sigma_R}{\mu_S + k_S \cdot \sigma_S} \qquad [80]$$

Where k_R e k_S are statistical factors selected for a desired probability of R being inferior to μ_R and S being superior to a μ_S, respectively. If a reliability index is defined as:

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{{\sigma_R}^2 + {\sigma_S}^2}} \qquad [81]$$

The probability of a failure, that is the probability of $R - S \leq 0$, is given by:

$$P_f = \Phi(-\beta) \qquad [82]$$

Where Φ is the cumulative distribution function of the standard normal distribution. If we go back to the central factor of safety, the reliability index becomes:

$$\beta = \frac{FS_c - 1}{\sqrt{{FS_c}^2 \cdot {V_R}^2 + {V_S}^2}} \qquad [83]$$

Where $V_R = \sigma_R / \mu_R$ and $V_S = \sigma_S / \mu_S$ are the coefficients of variation of the resistance and the loading, respectively. Subconsciously or not, almost every engineer from the early beginnings has this in mind. Call it caution. The central factor of safety may be rewritten:

$$FS_c = \frac{1 + \sqrt{1 - \left(1 - \beta^2 \cdot {V_R}^2\right) \cdot \left(1 - \beta^2 \cdot {V_S}^2\right)}}{1 - \beta^2 \cdot {V_R}^2} \qquad [84]$$

That shows, apart from everything else, the probabilistic dimension of what is sometimes despicably considered deterministic. The nominal factor relates to the central one as simply as:

$$FS_n = \frac{1 - k_R \cdot V_R}{1 + k_S \cdot V_S} \cdot FS_c \qquad [85]$$

And a different definition of the reliability index arises:

$$\beta = \frac{(FS_n - 1) + FS_n \cdot k_S \cdot V_S + FS_c \cdot k_R \cdot V_R}{\sqrt{{FS_c}^2 \cdot {V_R}^2 + {V_S}^2}} \qquad [86]$$

One may ask what the point of the above statements is, if the objective is discussing the occurrence of an undrained ground failure. The point is essential. Why do we use the factors of safety for? Among other objectives, the factor of safety is supposed to reflect the degree of uncertainty or random variability of resistance and loading (action or action effect). Going back to Figure 20, one may easily imagine decreasing the slope of the failure line (the resistance) and increasing the radius or moving the centre of the Mohr circle (the action or its effect) in such a way that each of them traduced a certain probability of exceedance of the respective variable. The problem of choice is still unanswered, but we may have found a rational argument to leave it like that. Perhaps we do need to know how bigger or heavier our old embankment dam could have been, in case we had constructed it in a couple of days, so that a foundation bearing capacity failure had occurred.

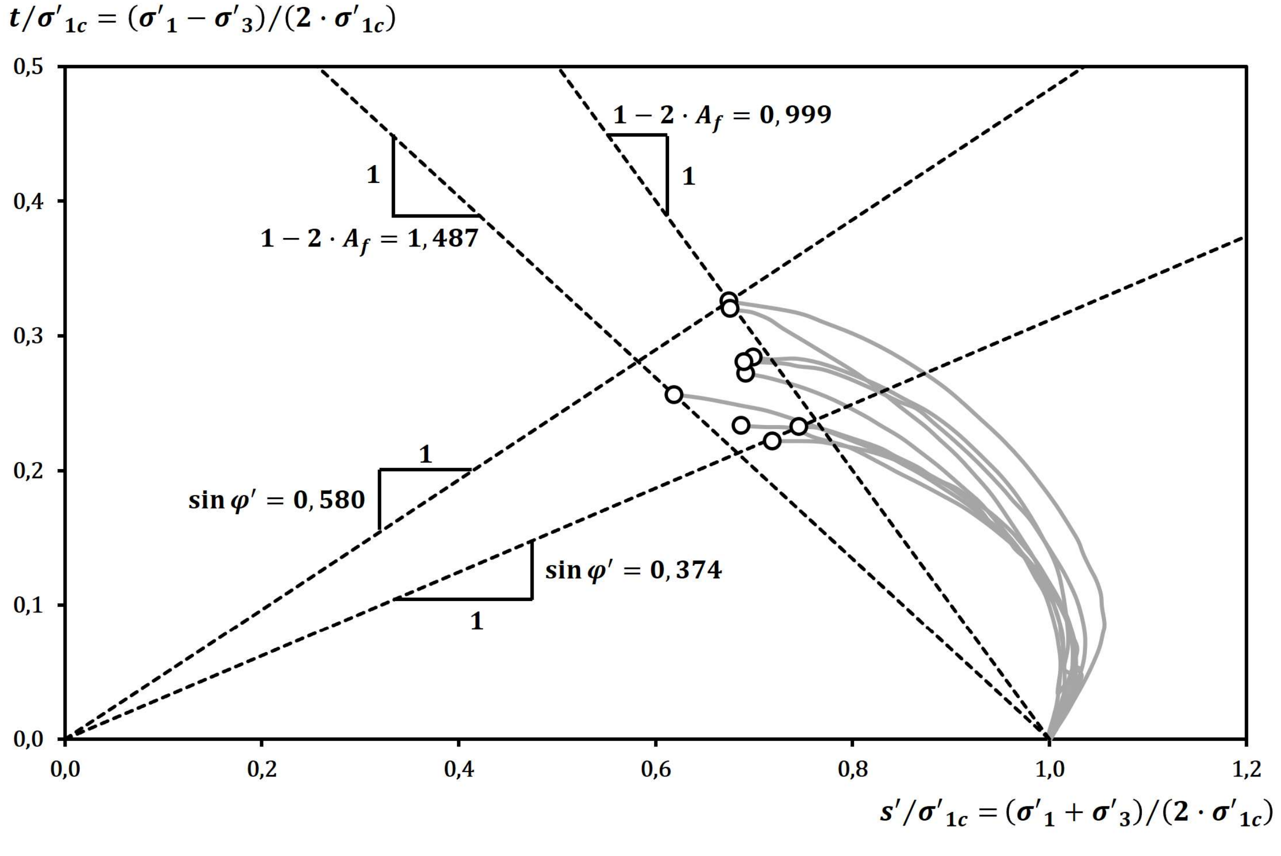

Figure 21

Another observation is allowed by this simplistic description of the probabilistic dimension of safety. The same value of factor of safety, whichever is its definition (FS_c or FS_n), may reflect a completely different probability of failure and even so it may be equally acceptable for two different works. Otherwise, depending on the consequences of a failure, different values of the factor of safety may be required for similar works. A simple example follows as an illustration.

Let us then see what values the factor of safety of a soil specimen should take so that the probability of its failure under an axisymmetric compression, in conditions of blocked drainage, did not exceed a target value. Data from nine CIUC triaxial tests performed on block samples of a clayey residual soil, as shown in Figure 21 on the previous page, will be used, but exclusively the specimens submitted to the same isotropic consolidation stress σ'_{1c}, which is deemed to be well beyond the mean pre-consolidation stress on site. It will be assumed that the load, $S = t$, applied by the laboratory technician is a deterministic variable and that the resistance, $R = s_u$, is a random variable.

The limit state function is:

$$g = R - S = 0 \Leftrightarrow g = s_u(\sin\varphi', A_f) - t = 0 \tag{87}$$

Where $\sin\varphi' = (t/s')_f$ and A_f are assumed to be random variables, independent of each other and following a normal distribution. Relevant data from tests is provided in Table 1.

Table 1

Test	$\sin\varphi'$	A_f	c_u/σ'_{1c}
1	0,312	1,047	0,232
2	0,339	1,171	0,233
3	0,393	1,066	0,272
4	0,483	1,000	0,326
5	0,308	1,134	0,222
6	0,474	1,006	0,320
7	0,414	1,244	0,256
8	0,407	1,029	0,284
9	0,407	1,052	0,281
μ (or $\mathrm{E}[\cdot]$)	0,393	1,083	0,270
σ (or $\sqrt{\mathrm{Var}[\cdot]}$)	0,063	0,083	0,037
V	0,161	0,076	0,139

Recalling equation 10, the undrained shear strength ratio can be expressed as a non-linear function of $\sin\varphi'$ and A_f, from where approximate estimates of the moments of its statistical distribution can be obtained as follows:

$$\mathrm{E}[c_u/\sigma'_{1c}] \approx \frac{\mathrm{E}[\sin\varphi']}{1+\left(2\cdot\mathrm{E}\left[A_f\right]-1\right)\cdot\mathrm{E}[\sin\varphi']} = \frac{0{,}393}{1+(2\times 1{,}083-1)\cdot 0{,}393} = 0{,}269 \tag{88}$$

For the mean value, and:

$$\begin{aligned}\mathrm{Var}[c_u/\sigma'_{1c}] &\approx \left(\frac{\partial s_u}{\partial\sin\varphi'}\right)^2\cdot\ \mathrm{Var}[\sin\varphi'] + \left(\frac{\partial s_u}{\partial A_f}\right)^2\cdot\ \mathrm{Var}\left[A_f\right] = \\ &= \left(\frac{\sin\varphi'}{\left(1+\left(2\cdot A_f-1\right)\cdot\sin\varphi'\right)^2}\right)^2\Bigg|_{\mathrm{E}[\sin\varphi'],\mathrm{E}[A_f]}\cdot\ \mathrm{Var}[\sin\varphi'] + \\ &+\left(\frac{2\cdot\sin^2\varphi'}{\left(1+\left(2\cdot A_f-1\right)\cdot\sin\varphi'\right)^2}\right)^2\Bigg|_{\mathrm{E}[\sin\varphi'],\mathrm{E}[A_f]}\cdot\ \mathrm{Var}\left[A_f\right] = \\ &= \left(\frac{0{,}393}{(1+(2\times 1{,}083-1)\cdot 0{,}393)^2}\right)^2\times\ 0{,}063^2 + \\ &+\left(\frac{2\times 0{,}393^2}{(1+(2\times 1{,}083-1)\cdot 0{,}393)^2}\right)^2\times\ 0{,}083^2 = 0{,}037^2\end{aligned} \tag{89}$$

For the variance. Both these values are practically equal to those calculated from the direct measurement of s_u (See the 4th column of Table 1). From equation 87, it is immediate that the variance of the limit state function, which represents a margin of safety, is the same as the undrained shear strength one.

Moving on to the factors of safety, let us stipulate that the nominal value of the undrained shear strength ratio, as used in equation 80, corresponds to a fractile of 5% of the statistical distribution and assume this to be a normal one. Also, let us assume that the statistics of undrained shear strength ratio from Table 1 would be the same as for an unlimited number of tests. This way:

$$\mathrm{Prob}(R\le R_k) = \mathrm{Prob}\left(\frac{R-\mu_R}{\sigma_R}\le\frac{R_k-\mu_R}{\sigma_R}\right) = \Phi\left(\frac{R_k-\mu_R}{\sigma_R}\right) = 5\% \tag{90}$$

Or:

$$\left(c_u/\sigma'_{1c}\right)_k \equiv R_k = \mu_R - K_R\cdot\sigma_R = 0{,}270 - 1{,}645\times 0{,}037 = 0{,}208 \tag{91}$$

If a probability of failure $P_f = 10^{-5}$ was specified, the reliability index would be $\beta = 4{,}265$ and the required nominal factor of safety should be:

$$FS_n = \frac{1 - k_R \cdot V_R}{1 - \beta \cdot V_R} = \frac{1 - 1{,}645 \times 0{,}139}{1 - 4{,}268 \times 0{,}139} = 1{,}89 \qquad [92]$$

And the central factor of safety should equivalently be:

$$FS_c = \frac{FS_n}{1 - k_R \cdot V_R} = \frac{1{,}89}{1 - 1{,}645 \times 0{,}139} = 2{,}46 \qquad [93]$$

That is, the deviatoric stress ($2 \times t$) that the laboratory technician could apply to a specimen should not exceed $2 \times 0{,}208/1{,}89 = 2 \times 0{,}270/2{,}46 = 0{,}22$ times the value of the isotropic consolidation stress. Proceeding this way the technician could expect that just 1 in 10.000 equal specimens, equally tested, were brought to failure. If the specified probability of failure was condescended by a 10-fold ($\beta = 3{,}719$) then the deviatoric stress could be limited, at the maximum, to 0,26 times the value of the isotropic consolidation stress. The example may seem risible, as the consequence of a specimen failure is nothing but anecdotal. However, the interest on the reliability subjacent to the margins of safety, expressed alternatively by equations 84 and 85, regains all its importance when extrapolated to the failure of the slope of a large embankment dam, with high consequences of failure. Such extrapolation implies the consideration of a plethora of sources of uncertainty, here surreptitiously left aside.

Nevertheless, for the analysis of a global ground failure an estimate with 95% confidence of the mean value of undrained shear strength available on site (statistical population), based on the mean value of the test sampling, should be used instead of a low fractile value. The statistical factor for the resistance would better be:

$$k_R = \frac{t_{n-1}(\alpha)}{\sqrt{n}} = \frac{1{,}860}{\sqrt{9}} = 0{,}620 \qquad [94]$$

Where n is the number of observations (9 in this case) and $t_{n-1}(\alpha)$ is the inverse of the left-tailed *t-Student* distribution, with $n - 1$ degrees of freedom and a confidence of $1 - \alpha$ (95% is used on equation 94). For the set of specimens considered before (Table 1), a 95% confidence estimate of the population mean value of the undrained shear strength ratio would be $\left(c_u/\sigma'_{1c}\right)_k \equiv R_k = 0{,}246$. This last value represents our best estimate of the real soil mean strength and occupies the place of the numerator in equation 80. Assuming, for simplicity, that the standard deviation of the population was the same as the sampling one, the new factor of safety, required to ensure the same probability of failure, $P_f = 10^{-5}$, would have to be raised to $FS_n = 2{,}85$ (compare with the value obtained in equation 92). One must recognise that the statistics of a sample may not be the statistics of the population.

Reasoning inversely, it can be concluded that for a fixed nominal factor of safety $FS_n = 1{,}30$, the probability of a specimen failure would be $P_f = 1{,}72 \times 10^{-5}$ ($\beta = 2{,}925$) and for a nominal factor of safety $FS_n = 1{,}50$, the probability of a specimen failure would be $P_f = 2{,}38 \times 10^{-4}$ ($\beta = 3{,}949$). The corresponding values of the central factor of safety should be $FS_c = 1{,}69$ or $FS_c = 1{,}94$, depending on the value of the specified nominal factor of safety. A reliability index lower than 3,3 would not be acceptable even for *agricultural buildings where people do not normally enter*, according to EN1990. An annual failure probability of 10^{-4} means that a failure event can occur once in 10.000 years, but, for the case of an undrained failure, time should be on the side of safety. The soil from which the block samples referred to on this section were collected occurs extensively, and to significative depths, on the foundation of a 90m high existing embankment dam, on the land where long-term undrained failures have become an alarming concern.

Illustrating further the role of uncertainty, the previous exercise will be repeated but using the test data from other specimens of the same nine triaxial tests that were consolidated to an isotropic stress four times lower than the one corresponding to data presented on Table 1 and Figure 21. For the new set of nine specimens, effective stress paths are shown in Figure 22 below. At this level of consolidation pressure, the specimens reveal the effect of the apparent overconsolidation of the residual soil formation. The statistics of the undrained shear stress ratio, for the sample constituted by these nine specimens, are a mean value $\mu = 0{,}445$ and a standard deviation $\sigma = 0{,}101$. The coefficient of variation is $V = 0{,}227$, which is 1,6 times greater than that obtained previously.

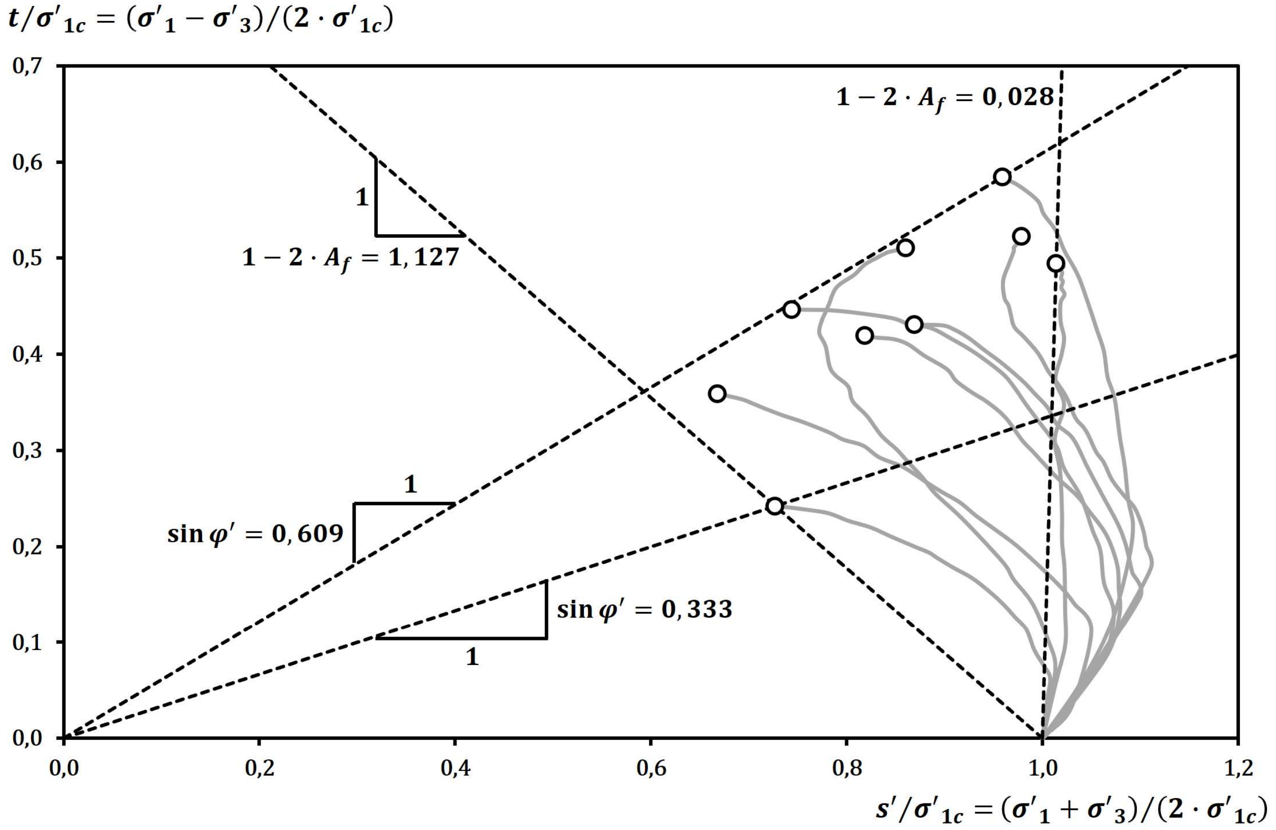

Figure 22

It is worth observing that, for this latter set of specimens, the coefficient of variation of $\sin\varphi'$ is the same as for the previous set ($V = 0{,}161$) but the coefficient of variation of A_f increased almost four times ($V = 0{,}283$). This highlights the magnified uncertainty of the undrained shear strength, in relation to the drained one. Among other factors, the difference on the variation of the undrained shear strength ratio for this latter set of specimens is explained by the erratic nature of the weathering process, mainly of chemical nature, of the local soil formation and to some extent the uncertainty related to the α_2 transformation factor is now introduced for the sake of discussion. It is also opportune to question if the adequate place for the excess pore pressures is on the side of soil resistance.

As before, we will keep admitting that the statistics of the sampling and the population are the same. To comply with a prescribed probability of failure given by $\beta = 4{,}265$, the required nominal factor of safety would have to be $FS_n = 18{,}90$ and the required central factor of safety would have to be $FS_c = 30{,}14$. We can interpret this, away from a theoretical point of view, as saying that difficulty any deviatoric load could be reasonably specified to ensure that practically none of the unlimited number of specimens that could be taken for the ground would fail. We must admit that in some circumstances, as is the case when the coefficient of variation of undrained shear strength is as high as 0,40, it is very unlikely that very stringent levels of reliability are achievable on the terrains of practice. Believing in the occurrence of long-term undrained failures is a serious consequential subject.

For this last set of specimens, if a fixed value of the nominal factor of safety was used, $FS_n = 1{,}5$, and applied along with a cautious estimate of the mean value of the population, using a statistical factor as indicated in equation 94, the probability of failure of a specimen from any other group of nine specimens tested equally would be $P_f = 1{,}03 \times 10^{-1}$ ($\beta = 1{,}264$) with a chance of 5% of being wrong. In the long run, the laboratory technician could expect that no more than 1 in 10 specimens was brought to failure if the maximum applied deviatoric load equaled 0,51 times the value of the consolidation stress.

There are no deterministic factors of safety, and we suspect that never really were in engineering practice. However, some codes of design require us to demonstrate that minimum fixed values of the factor of safety are attained as proof of safety, regardless of what. Considering that the coefficient of variation of the undrained shear strength is usually greater than the one exhibited by the drained shear strength, one would be forced to conclude that the applicable factor of safety would have to be equivalently greater for the case of a long-term undrained failure so that the reliability index was the same, as dictated by the importance of the work. Some fairness is introduced with the definition of a nominal factor of safety, as the different levels of uncertainty on different problems can be explicitly, although partially, contemplated, depending on the quantity and quality of the available information.

However, on the side of resistance, the factor of safety does not account only for un unfavorable deviation on the soil properties from its considered representative value, but also accounts for the randomness of the transformation factors, like α_1 and α_2, and the uncertainty of the resistance model of the ground used to verify the safety of a geotechnical structure. Therefore, in equation 80, a unit value cannot be used for the nominal factor of safety. In fact, the traditional minimum factors of safety, sometimes faced as deterministic entities, are to be used in the context of equation 80 and not in the context of equation 79. A different question is how the minimum required value of factor of safety, prescribed similarly in many design codes, has been used, if no guidance has been given for the conditions of its applicability. It would be regressive to use improved methods of analysis, capable of describing all complexities and overcoming all uncertainties, if the end prize was to fulfill the same exact requirement of a fixed minimum value.

4.4. A more “intelligent” safety format

On modern design codes, like the Eurocodes, based on the semi-probabilistic method, partial factors of safety are used. The safety condition is expressed as:

$$\frac{R_k}{\gamma_R} - \gamma_S \cdot S_k \geq 0 \tag{95}$$

Being γ_S the partial factor for loading and γ_R the partial factor for resistance. This is analogous to equation 80 by letting:

$$FS_n = \gamma_R \cdot \gamma_S \tag{96}$$

And defining the partial factors as:

$$\gamma_R = \frac{1 - k_R \cdot V_R}{1 - \beta \cdot \alpha_R \cdot V_R} \tag{97}$$

Where:

$$\alpha_R = \frac{\sigma_R}{\sqrt{{\sigma_R}^2 + {\sigma_S}^2}} \tag{98}$$

And:

$$\gamma_S = \frac{1 - \beta \cdot \alpha_S \cdot V_S}{1 + k_S \cdot V_S} \tag{99}$$

Where:

$$\alpha_S = -\frac{\sigma_S}{\sqrt{{\sigma_R}^2 + {\sigma_S}^2}} \tag{100}$$

According to the current version of EN1997, the partial factor for soil undrained shear strength is $\gamma_R \equiv FS_n =$ 1,4 and the partial factor for the drained shear strength is $\gamma_R \equiv FS_n =$ 1,25. On the pre-standard discussion period, a value $\gamma_R \equiv FS_n =$ 1,6 was first proposed for the undrained shear strength. Although the Eurocodes are not strictly applicable to the design of large dams with high consequences of failure, these values serve to illustrate the idea that a greater factor of safety is required for the analysis of an undrained failure. Apart from showing that all the definitions of factor of safety can be equally valid, another curiosity accrues from the calibration of the partial factors of safety carried for the Eurocode destined to geotechnical design, which was unable to depart radically from the historical and empirical old methodologies. At the end, most of dam slopes may still be considered safe when $FS_n \geq$ 1,5 under normal operation conditions, just depending on how much caution is injected into the definition of shearing resistance.

If, instead of trying to quantify how far from failure a soil element is under its actual state of stress, we simply attempted to determine if that same element was confidently far from failure, we would not need to make any assumption regarding the stress path to be followed up. The sufficiency of the separation between the actual state of stress and the state of failure would be provided by the partial factors of safety, covering for the uncertainties involved on the quantification of actions and resistances, as well on the models. Assuming that the true soil shear strength is given by the effective stress angle of shearing resistance, verification of safety of a soil element subjected to a compression in conditions of axial symmetry would resume satisfying the following inequality:

$$\frac{\tan\varphi'}{\sqrt{{\gamma_R}^2+\tan^2\varphi'}}-\frac{1-K}{1+K}\geq 0 \qquad [101]$$

Being $K={\sigma'}_3/{\sigma'}_1$, with the principal stresses corresponding to the actual state of stress. If pore water pressure and excess pore water pressures were acting on that soil element, their effect would simply be traduced by the effective stresses introduced on the stress ratio. In false triaxiality conditions, the second term of the first member of equation 101, for a specimen subjected simultaneously to an arbitrary increment of the total maximum principal stress $\Delta\sigma_1$ and to an arbitrary increment of the total minimum principal stress $\Delta\sigma_3$, would be:

$$\frac{1-K}{1+K}=\frac{({\sigma'}_{10}-{\sigma'}_{30})+(\Delta\sigma_1-\Delta\sigma_3)}{({\sigma'}_{10}+{\sigma'}_{30})+(\Delta\sigma_1+\Delta\sigma_3)-2\cdot\Delta u} \qquad [102]$$

Where ${\sigma'}_{10}$ and ${\sigma'}_{30}$ are the maximum and minimum effective principal stresses at the initial consolidated state, respectively. Increments of principal stress and the respective excess pore pressures, Δu, are not necessarily the ones required to bring the soil element to failure, rather ones resulting from the actual applied loading, $\Delta u=\Delta u(\Delta\sigma_1,\Delta\sigma_3)$. Under this "new" safety format, resistances stay on the side of resistances and actions stay on the side of actions. Excess pore water pressures, like pore water pressures or any other stresses in general, stay where the effects of actions seem to belong more appropriately. Curiously, the partial factor of safety for loading vanishes from the equations on the absence of a cohesion intercept ($c'=0$).

However, the Eurocodes do not preclude the need of a choice, once the existence of soil undrained shear strength is not negated. In terms of total stresses, the safety condition would be instead written as:

$$\frac{s_u}{\gamma_R}-\gamma_S\cdot\frac{(1-K)}{2}\cdot{\sigma'}_{10}\geq 0 \qquad [103]$$

With s_u calculated from equation 8. Assuming, for example, that $\varphi'=28°$, $A_f=1{,}1$ and $K_0=1-\sin\varphi'$, one could conclude that, using Eurocode's partial factors for soil resistance and admitting $\gamma_S=1$, just the safety condition in terms of effective stresses, according to equation 101, would be satisfied if no pore excesses were considered to be acting ($\Delta u=0$ in equation 102). For the left member of equation 101 to attain a value of 0, the partial factor of safety would have to be $\gamma_R=1{,}65$ ($>1{,}25$) and for the same to happen with equation 103 the respective partial factor would have to be $\gamma_R=1{,}34$ ($<1{,}4$). This simply results from not making or making assumptions regarding the effective stress path to failure, beyond that followed in consequence of the applied loading, $\Delta\sigma_1$ and $\Delta\sigma_3$.

Supposing that we were trying to assess the condition of a soil element on levelled ground at rest, equation 103 would strangely be telling us that the soil was already unsafe, whatever that means in philosophical terms. If that meant that no loading could be applied to the soil element, a gross error of appreciation was being committed, as that depended on the characteristics of an eventual future loading.

4.5. Total stress and effective stress analysis of undrained failures

It is common to read in some guidance documents published everywhere around the world that an undrained failure can be equally analysed in terms of total or effective stresses. For the latter type of analysis, it is also common to recommend the use of real measurements of pore pressures, including the excesses generated by loading. Obviously, if excess pore water pressures can be measured on the field, the following discussion applies to a short-term undrained failure and the pore pressures that can be measured are those generated by the applied loading, not those generated at non-existing failure. For instance, in the seventies and eighties decades of last century, several trial embankments were raised up until failure of the respective foundation occurred and both types of analysis compared. Some defended that, despite the limitations of both types, the effective stress analysis was better, especially because it relied on what we still admit being the true intrinsic resistance parameter of a soil, the angle of shearing resistance.

In line with equation 101, if the safety condition of a soil element was rewritten as:

$$\frac{\tan\varphi'}{FS} - \tan{\varphi'}_{mob} \geq 0 \Rightarrow \frac{\tan\varphi'}{FS} - \frac{\tau}{{\sigma'}_n} = \frac{\tan\varphi'}{FS} - \frac{\tau}{\sigma_n - \Delta u} \geq 0 \qquad [104]$$

Being $\tan{\varphi'}_{mob}$ the mobilised friction coefficient under a certain load, τ and ${\sigma'}_n$ the shear and normal effective stress acting on the plane of failure and Δu the excess pore pressure generated by the actual load, then we would not be making any assumption regarding the stress path that remained to be followed to reach failure. The first term on the left member of the inequality represents the resistance and the second one represents the loading, and both are independent of each other. A fallacy exists on affirming the equivalence of a total stress approach and an effective stress approach, as that is just true in a single situation, the onset of an actual undrained failure.

Let us think of a soil element subjected to a compression loading in false triaxiality conditions with blocked drainage. The initial consolidation state of stress (index 0) is represented by a maximum principal stress ${\sigma'}_{1,0} = \sigma_{1,0}$ and a minimum principal stress ${\sigma'}_{3,0} = \sigma_{3,0} = (1 - \sin\varphi') \cdot {\sigma'}_{1,0}$. We will assume that $\sin\varphi' = 0{,}48$. Also, we will assume that the pore pressure coefficient A grows linearly with the deviatoric stress and has a value of $A_f = 1$ at failure. From equation 9 we already know that the undrained shear strength is $s_u = 0{,}324 \times {\sigma'}_{1,0}$ and the shear stress at the failure plane, according to equation 13, is $c_u = s_u \cdot \cos\varphi' = 0{,}285 \times {\sigma'}_{1,0}$. This soil element will be subjected to an increment of maximum principal total stress $\Delta\sigma_1$ and an increment of minimum principal total stress $\Delta\sigma_3 = \Delta\sigma_1/3$, as dictated by the presumed external load configuration, but just up to a value of $\Delta\sigma_1$ that corresponds to a shear stress τ_1 on the failure plane equal to $c_u/1{,}2$, such that:

$$\tau_1 = \tau_0 + \frac{1}{3} \cdot \Delta\sigma_1 \cdot \sin(2\theta) = \left(\frac{1}{2} \cdot (1 - K_0) \cdot \sigma_{1,0} + \frac{1}{3} \cdot \Delta\sigma_1\right) \cdot \sin(2 \cdot \theta) = \frac{c_u}{1{,}2} \Rightarrow \qquad [105]$$

$$\Rightarrow \tau_1 = 0{,}237 \times \sigma_{1,0} \Rightarrow \Delta\sigma_1 = 0{,}091 \times \sigma_{1,0}$$

Where θ is the angle between the failure plane and the maximum principal stress plane. Following equation 1, the excess pore pressure generated by the total stress increments is $\Delta u = 0{,}081 \times \sigma_{1,0}$ for a pore pressure coefficient $A = 0{,}83$. The total normal stress acting on the failure plane following the application of the stress increments is:

$$\sigma_{n,1} = \sigma_{n,0} + \frac{1}{3} \cdot (2 + \cos(2\theta)) \cdot \Delta\sigma_1 = \\ = \left(\frac{1}{2} \cdot (1 + K_0) \cdot \sigma_{1,0} + \frac{2}{3} \cdot \Delta\sigma_1\right) + \left(\frac{1}{2} \cdot (1 - K_0) + \frac{1}{3} \cdot \Delta\sigma_1\right) \cdot \cos(2 \cdot \theta) = 0{,}691 \times \sigma_{1,0} \qquad [106]$$

Substituting the resulting values on equations 105 and 106 into equation 104, we can conclude that a factor of safety $FS = 1{,}41$ could still be applied to the effective stress shear strength parameter to verify the safety condition for an undrained loading. If equation 74 was used, the factor of safety would obviously be $FS = 1{,}2$. This simple conceptual example shows that equation 74 and ultimately a total stress analysis implies the excess pore pressures not being those generated by loading, rather ones much closer to the value that would be eventually generated, following a fixed stress path, if the real loading was increased until failure occurred. The use of real field measurements of pore water pressures or alternatively a prediction of them, as used in this example, do not satisfy the safety requirements embodied by the new “paradigm”, discussed on next chapter.

For a factor of safety $FS = 1{,}2$ to be obtained based on equation 104, the excess pore water pressure would have to be $\Delta u = 0{,}171 \times \sigma_{1,0}$, corresponding to a pore pressure coefficient of $A = 0{,}92$. The maximum principal stress increment to reach that level of excess pore pressure would have to be $\Delta\sigma_1 = 0{,}171 \times \sigma_{1,0}$, which is 1,98 times greater than the one applied by the real loading. Therefore, a total stress analysis is incongruent if an attempt is made to isolate the true soil resistance and the external loading or its effects. So, when importing to a limit equilibrium model the pore pressures obtained with a stress-deformation analysis, eventually including the effect of ongoing consolidation, please remember that, at least from someone´s points of view, we may be heavily violating the safety requirements.

If partial factors of safety were used instead, as discussed on subchapter 4.4, equation 104 could eventually be modified as follows:

$$\frac{\tan\varphi'}{\gamma_R} - \frac{\tau}{\sigma_n - \gamma_S \cdot \Delta u} \geq 0 \qquad [107]$$

Where a partial factor $\gamma_S = 2{,}12$ would be required, accompanying $\gamma_R = 1{,}2$, so that the effective stress analysis produced the same result as the total stress analysis. This is in violation of the principle of single source, once two effects of the same action were being factored differently.

Otherwise, if a total stress analysis of an undrained failure on the short-term is preferred, due to the difficulty of predicting the pore water pressures at failure, and if the undrained shear strength is represented as a ratio in relation to the mean consolidation stress, some care must be also exercised. Continuing with the same example, the undrained shear strength ratio could be written as:

$$\frac{s_u}{p'_0} = \frac{0{,}324\ \cdot \sigma'_{1,0}}{\frac{1}{3} \cdot (1 + 2 \cdot K_0) \cdot \sigma'_{1,0}} = 0{,}477 \qquad [108]$$

And the available undrained shear strength would be:

$$\begin{aligned} s_u &= \alpha_2 \cdot \left(\frac{s_u}{p'_0}\right) \cdot (p'_0 + \Delta p') = \alpha_2 \cdot \left(\frac{s_u}{p'_0}\right) \cdot (p'_0 + \Delta p - \Delta u) = \\ &= \alpha_2 \cdot \left(\frac{s_u}{p'_0}\right) \cdot \left(p'_0 + \frac{2}{9} \cdot (1 - 3 \cdot A) \cdot \Delta\sigma_1\right) \approx 0{,}324 \times \sigma'_{1,0} \end{aligned} \qquad [109]$$

Considering $\alpha_2 \approx 1{,}03$, as calculated with equation 69, to compensate for a slight reduction of the initial mean effective stress resulting from the applied loading. This may seem trivial, but we may know of a pair of cases where large embankment dams founded on a weak compressible foundation were declared as having a global factor of safety lower than 1,3 at the end of construction, being Δu on equation 94 equivocally taken as null. That value is the long-term factor of safety, not the short-term one.

4.6. The loading rate

Up to now, we have been addressing the factor of safety against an undrained failure from the perspective of the insufficiency of the applied loading to cause a failure, just being able to bring the soil closer to it. The distance separating the actual stress state, to where the soil was conducted by the loading, from one upon many possible states of failure has been previously faced as a question of choice. In part, the choice is the same that the laboratory technician is forced to make ahead of shearing a specimen on the triaxial apparatus. But, in the real world, the tap cannot be closed, and the soil is allowed to drain while it is being sheared. How much the soil drains will depend on how fast the loading is applied to it. The rate of loading will mark the eligibility of any stress path.

Let us suppose that a soil specimen could be subjected to a compression in false triaxiality conditions under different load rates, being that way able to dissipate part of the pore pressure excesses while it was being sheared. For a sufficiently slow loading rate the deviatoric stress at failure, q_{Df}, would be maximum (drained strength) and for a sufficiently fast loading rate the deviatoric stress at failure, q_{Uf}, would be minimum (undrained strength). In between, under a generic loading rate $\dot{\sigma}$, the deviatoric stress at failure (index f for failure is omitted for simplicity) would be:

$$\frac{q(\dot{\sigma}) - q_U}{q_D - q_U} = 1 - \frac{\Delta u(\dot{\sigma})}{\Delta u_U} = \overline{U} \qquad [110]$$

Where $\Delta u(\dot{\sigma})$ is assumed to be the average undissipated excess pore pressure at failure corresponding to the loading rate under analysis and Δu_U is the maximum average undissipated excess pore pressure at failure, corresponding to the fastest loading rate. Thus, we are admitting that $\Delta u(\dot{\sigma})$ is uniformly distributed on the failure planes of the specimen, even knowing that not to be true. $\overline{U}$ is the average degree of consolidation of the specimen at failure. From equation 110, the secant effective path to failure can be fully defined by a rate-dependent pore pressure coefficient:

$$A(\dot{\sigma}) = \frac{(3 - M_c) \cdot (1 - \overline{U}) \cdot A}{3 + (3 \cdot A \cdot \overline{U} - 1) \cdot M_c} \qquad [111]$$

Where M_c is the slope of the failure line on the p'- q stress space and A is the pore pressure coefficient at failure for the fastest loading rate (undrained loading). If $\overline{U} = 1$ then $A(\dot{\sigma}) = 0$ and if $\overline{U} = 0$ then $A(\dot{\sigma}) = A$, as expected. Recovering Figure 20, equation 111 demonstrates that, when the tap is left open, there are infinite effective stress paths to failure in between the two ideal limits corresponding to the standard triaxial tests (CIUC or CIDC). Even if as a gross approximation, the theory of one-dimensional consolidation may provide us a relationship between the average degree of consolidation and the loading rate. For a constant stress rate of loading, we have:

$$\dot{\sigma} = \frac{d\sigma_1}{dt} = \frac{\Delta\sigma_1}{\Delta t} \qquad [112]$$

Where Δt is the interval of time required to bring the specimen to failure ($t_f = t_0 + \Delta t = \Delta t$) from its initial isotropic consolidation state and $\Delta\sigma_1$ is the maximum principal total stress increment required for the same purpose, such that $q_f = q_0 + \Delta q = \Delta\sigma_1$. For a constant stress rate loading, the average degree of consolidation relates to the time interval to failure as follows:

$$\overline{U} = 1 - \frac{32}{\pi^4 \cdot T} \cdot \sum_{m=1}^{\infty} \left(\frac{1}{(2 \cdot m + 1)^4} \cdot \left(1 - \exp\left(-\frac{\pi^2}{4} \cdot (2 \cdot m + 1)^2 \cdot T \right) \right) \right) \qquad [113]$$

Where T is a dimensionless time factor:

$$T = \frac{c_v \cdot \Delta t}{H^2} \qquad [114]$$

Being c_v the coefficient of consolidation, considered constant throughout the process, and being H the length of the drainage path.

To exemplify, let us consider a 0,2m high specimen of a soil having a coefficient of consolidation c_v =10^{-7}m^2/s, a shearing resistance corresponding to $M_c = 1{,}143$ and a pore pressure coefficient at failure $A_f = 1{,}1$ under conditions of blocked drainage. Under free conditions, suppose that drainage is just possible at one of the specimen's ends, from where $H = 0{,}2$m in equation 114. To reach a 95% average degree of consolidation at failure, a constant stress rate $\dot{\sigma} = 0{,}482$kPa/h should be applied, and the corresponding pore pressure coefficient would be $A(\dot{\sigma}) = 0{,}019$ at failure. A tenfold increase on the stress rate, such that $\dot{\sigma} = 4{,}82$kPa/h, would result in a degree of consolidation $\overline{U} = 50{,}6\%$ at failure and a pore pressure coefficient $A(\dot{\sigma}) = 0{,}268$.

Another tenfold increase on the stress rate, such that $\dot{\sigma} = 48{,}2\text{kPa/h}$, would result in a degree of consolidation $\overline{U} = 12{,}7\%$ at failure and a pore pressure coefficient $A(\dot{\sigma}) = 0{,}763$.

For the above exemplification, the loading rate was assumed to be the rate of application of the maximum principal stress, $\dot{\sigma} = \dot{\Delta\sigma_1}$, which, as hypothesised, is proportional to the rate of pore pressure generation with a constant pore pressure coefficient equal to the one at failure. Strictly sense, equation 113 applies for constant rate of pore water pressure generation as can be observed from the one-dimensional consolidation differential equation, considering the pore fluid and the soil grains to be incompressible:

$$\frac{k}{\gamma_w} \cdot \frac{\partial^2 \Delta \mathrm{u}}{\partial z} = \frac{\partial \Delta \varepsilon_v}{\partial t} \qquad [115]$$

Being k the soil permeability and ε_v the volumetric strain. An increment of volume change can be described by:

$$\Delta\varepsilon_v = -C \cdot (\Delta p' + \alpha \cdot \Delta q) = C \cdot \Delta u - C \cdot (\Delta p + \alpha \cdot \Delta q) \qquad [116]$$

Where C is a compressibility parameter. Letting $\alpha = A - 1/3$ in equation 116 and substituting on equation 115:

$$\frac{k}{C \cdot \gamma_w} \cdot \frac{\partial^2 \Delta u}{\partial z} = \frac{\partial \Delta u}{\partial t} - A \cdot \frac{\partial \Delta \sigma_1}{\partial t} = \frac{\partial \Delta u}{\partial t} - \frac{\partial \Delta u_U}{\partial t} \qquad [117]$$

Where Δu_U is the excess pore pressure generated by the load increment $\Delta\sigma_1$ in conditions of blocked drainage. Defining the multiplicative constants on the first member of equation 117 as the consolidation coefficient c_v and assuming a constant rate of pore pressure excess generation, $\dot{u}_e$, we finally have the differential equation that should govern the problem under analysis:

$$c_v \cdot \frac{\partial^2 \Delta u}{\partial z} = \frac{\partial \Delta u}{\partial t} - \dot{u}_e = \frac{\partial \Delta u}{\partial t} - A \cdot \dot{\sigma} \qquad [118]$$

From the solution of which, apart from the average degree of consolidation advanced in equation 113, the undissipated excess of pore pressure could be calculated at any time and position along the specimen, $\Delta u(z, t)$.

Figure 23, on next page, shows the theoretical dimensionless consolidation curve for the case of cylindrical specimen, with a radius of half the height, allowed to drain simultaneously for both the ends and for the radial boundary. Over the curve, the results of a series of triaxial CIUC tests, performed with different loading rates on specimens of the same material consolidated to the same isotropic stress, are plotted to show that the above theoretical cogitations are not as far from reality as the underlying assumptions would suggest. The referred test data was published by Bishop and Henkel, by the end of the fifties of last century, while establishing minimum values for the strain rate to be used on standard CID triaxial tests.

This section has been another attempt to show that, in concept, soil responds to loadings, in the case depending on the rate of its application. Even if just shallowly, we are restoring to real problems the partially drained response of real soils to real loads, ranging between the extreme responses measured on standard laboratory tests. Moreover, in what concerns to undrained failures on the long-term, if the actual loading is being applied at a null rate, $\dot{\sigma} = 0$, it seems incongruent to define a factor of safety by choosing an effective stress path that corresponds to the pore pressure coefficient that would be manifested at failure if a very fast loading rate was being applied instead. Nevertheless, a choice still needs to be made, and the best choice does not necessarily have to be congruent, rather one that provides good practical results, including economical results. From what we have seen on this chapter, it seems that there is no indisputable theoretical argument to support a choice. We fully trust that our grandfathers in science already knew it, and more.

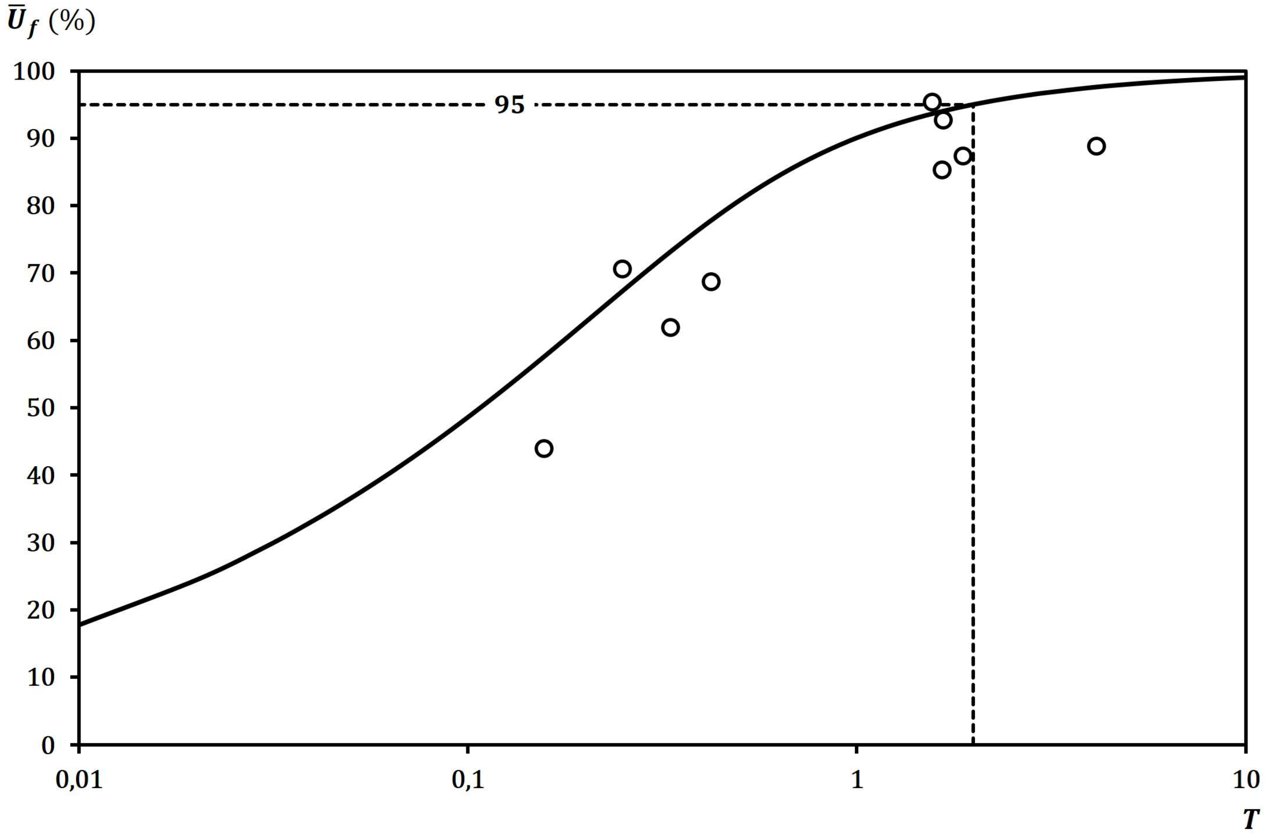

Figure 23

5. PARADIGM SHIFT

5.1. The old “paradigm”

On a real problem, one that is not a laboratory experiment, there are no taps to be closed, nor open. There are loadings, timings for loadings and loading rates. Soil will respond to loadings as dictated by its own drainage capacity, either generating a great amount of excess pore pressures or none, either dissipating them slowly or quickly. In due course, there is a time when loading and response are steadily equilibrated, and the excess pore pressures eventually generated earlier by that loading have vanished. That is the time when the loading rate is null and it is a time, like any other time, when theoretical propositions can be observed and measured on a real structure. That time used to be known as the long-term in soil mechanics.

As pictorially explained by Bishop and Bjerrum, on early beginning of the sixties of last century, classical soil mechanics proposed a pragmatic, yet inestimably precious, methodology to solve geotechnical problems like the stability of an embankment dam founded on weak compressible soils. We can estimate the shear strength of soil under two extreme and ideal drainage conditions, as reproduced by testing. We can recognize the timing, and the rate of loading applied to the ground and the bounds of the soil response to it. If safety is verified for the two bounds of soil response according to the timing of the loading that produces it, safety will, according to the old "paradigm", be satisfied at all instants along the lifetime of a structure, as long new loadings are not applied. Obviously, this methodology, distinguishing a short and a long term, just makes any sense if the problem involves soils that generate positive excess pore pressures when loaded and dissipate them afterwards.

Accordingly, for a sufficiently old existing dam, already proved by all permanent and transient loading conditions to which it has been design, there seems to be no reason to travel back to the past and reanalyse its stability under conditions that no longer exist, namely the critical ones existing at the end-of-construction. For a dam living on the long-term of soil mechanics, despite how much excess pore pressures the materials involved once may have generated, or might generate on the future if a new loading comes into play, slope stability analysis used to be conducted in terms of effective stresses (φ'), without considering any other pore pressures that not those due to steady state seepage that the dam should be prepared to cope with. This was the choice made for the purpose of defining a factor of safety, one as licit as the choice of any other stress path to failure from those available. Naturally, that was not the most pessimistic choice possible to put forward initially but is a choice that survived over half-century of repeated validation. As far as we know, and we confess to knowing little, there has not been yet an undrained failure in the long-term. Excessive pessimism is a luxury that few can afford.

For the analysis of stability of the downstream slope of an existing embankment dam under normal operation conditions, the old "paradigm" stated:

$$\Delta u = 0, \ \ \forall \Delta u \tag{119}$$

Being this condition applied to all soils regardless of their state and drainage capacity, with no exceptions. The minimum required factor of safety used to be $FS = 1{,}5$, a value that for most, not all, existing dams, competently constructed and conserved, still warrants an adequate level of structural reliability, or a sufficiently low probability of failure. We remember that in subchapter 4.2 it was demonstrated that a slope stability analysis by the limit equilibrium method, conducted with effective stresses, always includes some amount of excess pore pressures. The "old" paradigm is completely consentaneous with the elemental concept of soil response dependency on loading, including its composition, intensity and rate. It is also consistent with the distinction between resistances and actions, assuming that we are able to distinguish different loading situations and the exceptionality of their occurrence throughout the life of a structure.

It is true that, prior to the year of 2019, some guidance documents of some respectable supervising agencies, despite lecturing about the differences between undrained and drained soil responses and the loading conditions for their occurrence, shyly opened a footnote exception and that exception unruled the plausibility of drained responses for saturated contractile soils. From a practical point of view, it was a waste of lecturing, once distinguishing both types of soil response just makes any sense because of the exception itself. If soils simply behaved according to their state and regardless of loading, positive excess pore pressures would be simply faced as a material property for those soils able to generate them. Those agencies were blazing the trail for the arrival of a new "paradigm".

5.2. The new "paradigm"

Although having followed the interminable discussions about the validity of the old "paradigm", we did not have a good feeling about the consequences of the new one, up until the moment, in 2019, when we arrived at a land, some long thousands of kilometers away from home, where we were presented with the cataclysmic vision of dozens, if not hundreds, of existing embankment dams being judicially interdicted because of the danger of spontaneous undrained failures. Loads had been replaced by "triggers" and "triggers", on the contrary of actions, can be almost anything we wish to imagine, even left unpronounced. The new "paradigm" is as easy to explain as the old one, being just another choice. On our best interpretation, the new "paradigm" states:

$$\Delta u = \max\begin{pmatrix}\Delta u & , & \Delta u > 0\\ 0 & , & \Delta u \leq 0\end{pmatrix} \qquad [120]$$

Which seems obvious but is not as obvious as it seems. Uncoupling excess pore pressures into two components, as in equation 22, provides, erroneously, the idea that one of them, Δu_p, is dependent of an active loading, while the other one, Δu_q, is a kind of material behavioural property, from which the soil cannot escape. Therefore, a distinction between the short-term (ST) and the long-term (LT) needs to be reintroduced:

$$\Delta u = \begin{cases}\Delta u_p + \Delta u_q & \text{for} \quad ST\\ \Delta u_q & \text{for} \quad LT\end{cases} \qquad [121]$$

This applies to all soils in any state and to any loading conditions under analysis, including no new loadings being applied. In summary, the new "paradigm" replaces soil response to loading by soil behaviour, which for some soils is the one manifested on blocked conditions of drainage or, presumably, under an excessively fast active loading rate on the field, despite the actual loading rate being slow or even null. For other soils, inversely, the soil behaviour is the one manifested on free drainage conditions, once if sheared too fast their strength will be greater. Both types of soil, with their own behaviour, may coexist on the same problem. The shear strength of a soil becomes unique, the lowest of all possible on the state the material is encountered.

Our aim is to discuss the slope stability of an embankment dam founded on soft soils at the long-term, when all excess pore pressures generated by the actual loading have been dissipated. The analysis of such situation in terms of total stresses has been dealt on Chapter 2 and Chapter 3, which, in the case of a limit equilibrium analysis, has the advantage of omitting estimates of excess pore pressures. Equal results (FS) may be achieved following an effective stress approach, for which it is required that in any soil element along a potential slip surface an artificial excess pore pressure is considered:

$$\Delta u = \sigma_{n0} - \frac{s_u}{\tan\varphi'} \qquad [122]$$

Where σ_{n0} is the total normal stress acting on the plane of failure. The value of the excess pore pressure calculated this way just intends to provide an operational value of effective stress, one that makes the drained shear strength to be equal to the expected undrained shear strength, without needing to consider explicitly any change of total stress. For the long-term situation, this excess pore pressure value is a fraction of the one that would be generated by the deviatoric component of loading, if the latter had been increased up to failure. If a specimen of soil subjected to a compression loading on the triaxial apparatus, with a total stress path defined by $\Delta\sigma_3 = -\Delta\sigma_1$, is used as example, equation 122 becomes:

$$\Delta u = \left(\Delta u_q - \Delta\sigma_n\right) - \frac{(1-\cos\varphi')\cdot s_u}{\tan\varphi'} = \left(2\cdot A_f - 1 + \sin\varphi'\right)\cdot\Delta\sigma_1 - \frac{(1-\cos\varphi')\cdot s_u}{\tan\varphi'} \qquad [123]$$

Where it can be seen, namely on the first term of the last member of this equation, that a positive excess pore pressure would really exist at failure on the test because an increment of deviatoric loading ($\Delta s = \Delta\sigma_1$) had effectively been applied, not because any reason of behavioural nature. Whatsoever, the new “paradigm” has decided that, in a real problem and on the contrary of the test given here for example, a fictitious excess pore pressure will be acting on the soil even if no deviatoric stress increase results from loading. It has also decided that there is just one component of the excess pore pressures that can dissipate as the consolidation process evolves, while the other one remains quiescent.

Despite the apparent simplicity of this new choice, we have seen throughout this booklet that many difficulties arise when trying to put it into practice. First, the sought lowest resistance depends inevitably on some assumption regarding the loading that could mobilise it and that goes well beyond the assumption of a permanently closed tap in nature. Second, thinking by a moment of the factor of safety as a load factor, we will be trying to demonstrate that the ground can resist a certain amount of increased loading, admitting that a part of its effects, the pore pressure excess, grows disproportionately to the other one, the total stress. It is a choice, not a consequence of some law of physics.

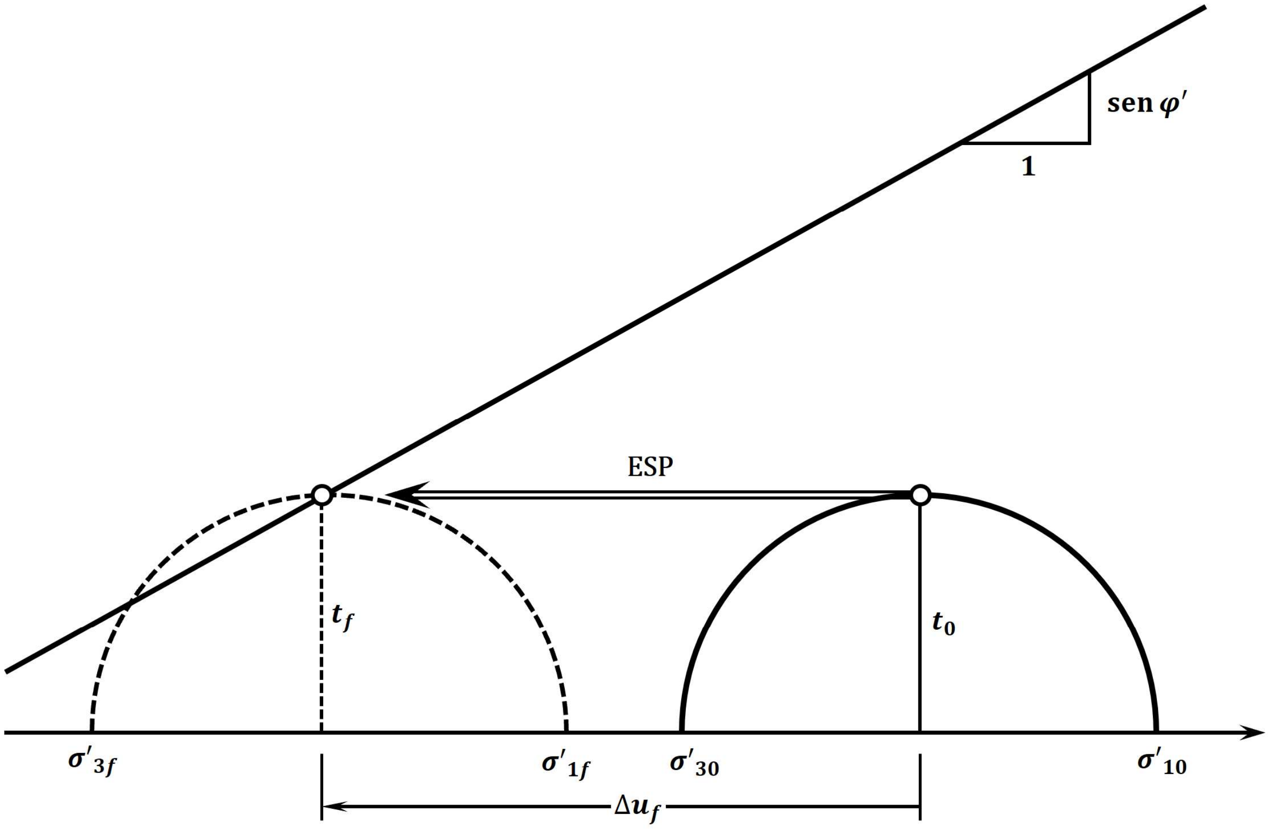

Figure 24

So, under the pledges of the new "paradigm", there is absolutely no point in distinguishing drained or undrained response conditions nor to associate them to loading situations along the lifetime of a structure like an embankment dam. Contractile soils never drain, and dilatant soils always drain, that is what is supposed to be accepted. Nevertheless, the short-term and long-term of soil mechanics maintains all its actuality, because consolidation is still recognised to be a real phenomenon, and the undrained shear strength is still supposed to increase with time, during and after the load application.

Figure 24, on previous page, shows the effective stress path of the wettest of all conceivable failures, though one that is not undrained. An element of soil was subjected to an increase of pore water pressure, Δu_f, and moved from the actual state of stress (index 0) to a failure state (index f) without any other external loading being applied. This could be the case of a soil element on an embankment submitted to a raise of the saturation surface. The fact that the saturation surface can hypothetically raise in any dam does not mean that we are condemned to consider that the shear strength of soil is not more than that corresponding to the failure state onto which the soil would be taken if the increase of pore pressures was sufficient for failure to occur. The same argument can be paralleled with the one that defends that the excess pore water pressures to be considered are those generated by loading, as in this case could be any real increase of the free water loading, $\Delta u < \Delta u_f$, insufficient to cause a failure. This last example is going up on the top list of "triggers" for undrained failures, even if there are no excesses of pore pressure building up.

5.3. The shift

Going through this subject of soil undrained shear strength and ground undrained failures, we just found what supposedly was found before by everyone else or at least by someone else. Engineering is also made up of choices and validation of choices. Plausible reasons for a "paradigm" shift in geotechnical engineering, such as this retrogressive and costly one, are unlikely to be found in soil mechanics. Try not leaving the triaxial apparatus tap closed or try interrupting a laboratory test before the specimen fails and the lost world will be recovered, the one where the old "paradigm" was first enunciated.

Some recent cases of progressive failure in strain-softening soils and the inadequate consideration of strain-compatibility between different soils intervening on the same problem or between the different simultaneous modes of failure occurring on the same soil are being abusively used as arguments for the announced shift, but inasmuch what an undrained failure may be interpreted to be, those cases prove precisely the contrary. Cumulatively, it is becoming popular to affirm that undrained failures are sudden by nature, unannounced to men and unpredictable by machines, what, as a first reaction, are essentially bad news for the monitoring and the computing businesses. For the assessment of the safety condition of a slope, it does not seem too decisive to know how fast the debris can move after a failure, nor does it seem that the speed of a failure is a problem avoidable by shifting the "paradigm". Perhaps is a problem prevented with better safety, looking into the consequences of a failure. Here seems to be some confusion regarding the mechanisms of structural strength loss or brittleness and the reduction of effective resistance due to generation of excess pore pressures by loading (and allied deformation). Both are not mutually exclusive, not necessarily inclusive.

The oddest argument that we could discover while trying to expand our library of beliefs stated that there are excess pore pressures generated by loadings and then, over and above them, there are excess pore pressures generated by shearing, as if shearing and loading were actors of the same play, mistakenly playing at different stages. This is not truly an argument, but the description of the "new" paradigm on its own. We have tried to make clear that it is generally on the engineers' primal interest to keep structures away from failure, within a convenient mobilisation of soil strength, and to less extent is of general interest to know what happens beyond the unmeasurable state no one desires to see.

The justifications given for the ongoing shift seem condescending, and its necessity lacks supporting evidence. Why do we need a shift? Where and when has the old "paradigm" failed us? Have you seen an undrained failure in the long-term? The new "paradigm" may bring more safety, but not necessarily better safety. For sure, it brings a more extravagant safety, should we need it and could we pay for it. Even so, we admit that better arguments may yet lie somewhere under the mountain of dust that still covers our library.

6. EPILOGUE

On the contrary of good technical books, this booklet requires the author to present some sort of closure. The land to which the journey took the author really exists on the world map and five years of his life were spent wandering from one existing old embankment dam to another old existing embankment dam. On his arrival, all those earth structures were about to become, on the minds of the local reputed colleagues, threatened by sudden foundation undrained failures, events that could be caused by any among many nameless "triggers". On his departure, few, if any, of dozens of structures remained free from blame and the accusation was irresistibly unanimous for everyone, or nearly. Few may claim having, like the author had, the opportunity to set foot on so many endangered embankment dams, some of them with slopes that were miraculously standing beyond the limit of equilibrium ($FS \ll 1$). Less than few may claim having assisted from a balcony to a "paradigm" shift, accepted by so many in so little time and finally becoming law by decree of lawmen. Contractile and undrained become synonyms in a couple of years. Figure 1, on the first page of this booklet, should be amended in accordance, erasing from it the mysterious effective stress path achieved with the tap open.

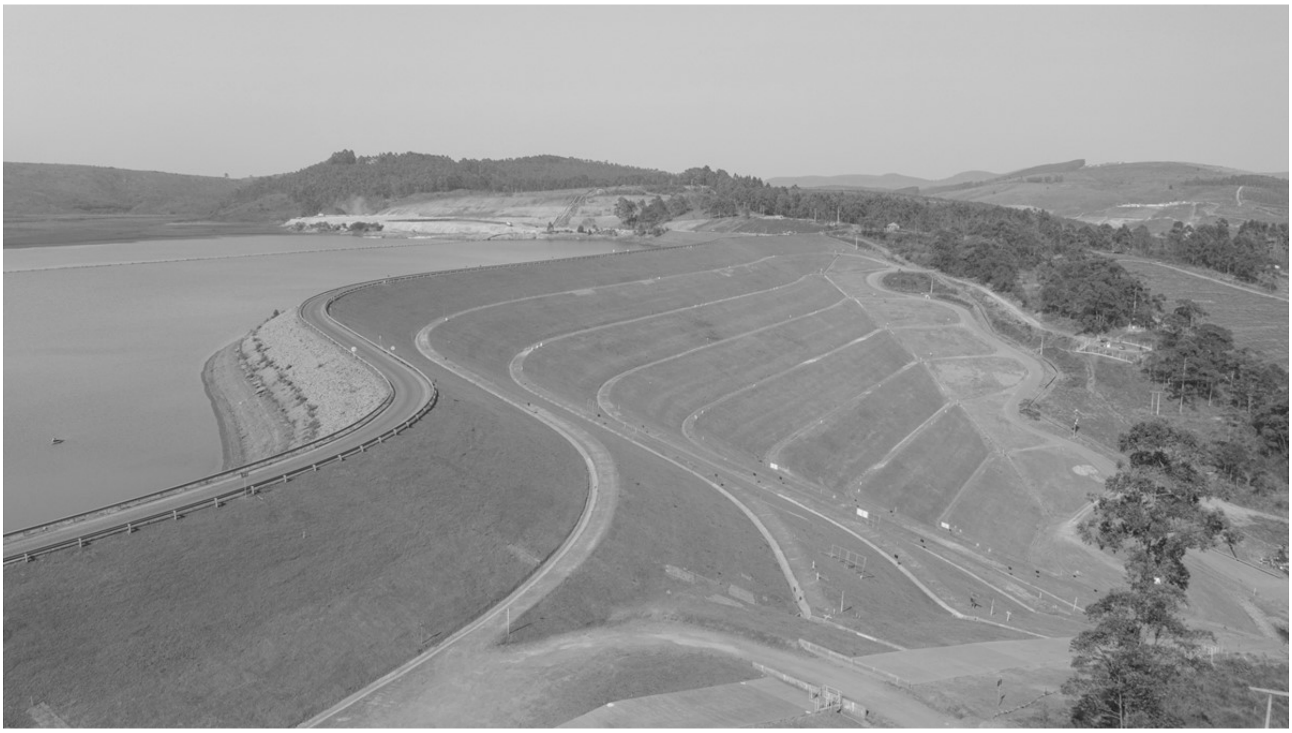

Figure 25

On some cases, the soils susceptible to such dangerous "behaviour" occurred in minuscule spots of transported soils overlying solid foundation materials and on other cases occurred in equally minuscule and erratic spots within monstrous packs of decomposed rock. Figure 25, at the end of previous page, shows a "marvelous" embankment dam that exists no more as can be seen on a photograph from the end of 2022. A minuscule spot of colluvium occurring at the toe of the slope on the left abutment was decisive for the dam to fall from grace, almost a decade after its construction being completed. Do not need to ask, because we will tell in advance that no sign of structural distress was ever observed. As the colluvium was contractile and contractile soils are known not to drain, the dam was lowered, reprofiled and buttressed. In between, some dozens of piezometers were installed on the medium stiff alluvium spot to measure what no longer could be measured, the excess of pore water pressure eventually generated one decade earlier while the earthworks processed. But that, contractility, was enough as proof. The "new" paradigm gained hordes of followers, some fervorous, and no dam, like this one, would be left in peace thereafter.

"Paradigm" shifts are said to take decades to be accepted, the time necessary for indoctrination of new generations, admitting that new generations can be indoctrinated. Nevertheless, for a "paradigm" shift to occur rapidly and nonviolently, it would be expectable that the reasons behind it were intelligibly explained by its proposers or advocates. They were not. Like the wind, belief blows where it wishes. During this journey, no explanation was even sought, and the local technical community adhered to the new "paradigm" in a craze, including the ones paid to assess safety, the ones that paid for the assessment of safety and those that oversaw the safety assessment activity. A mass conversion to the creed of unsafety acted like a dust storm sweeping the author's library of beliefs. Contemplating such a level of enthusiasm, why should someone want to be alone on the other side of barricade? As resources are unlimited, why should there be barricades? At the end, all this is just engineering…

What we know for sure is that all those safety assessors and activists were the same that a couple of months earlier to the beginning of the shift, and for decades until then, had been professing their faith in the old "paradigm" and swearing the eternal safety of the structures they had designed and built, over contractile soils because no others existed to place their dams above. The word "undrained" had never been written before on any technical report across that land and the author, to allow affirm it, read hundreds after hundreds of historical documents. While everything was crumbling down, surprisingly, there was no regret or sorrow, not even a small remembrance of the previous creed. A simple hail to the new "paradigm" and happiness for the mountains of rock available to build buttresses. The ease with which fundamental choices can be dramatically changed without plausible reason is what, at the very end, truly confronted the author´s beliefs, not in soil mechanics, but on his profession. What a profession this may be that allows us to declare unsafe today what we solemnly declared safe yesterday and get out of it like a redeemer or a savior.

black ribbon free publications